EAE 130A Project 2 Report:

Airplane Drag and Performance Analysis

Emre Mengi

ID: 913707050

University of California, Davis

2020

Abstract

This project is concerned about analyzing the drag characteristics and performance values of a Cessna 152 aircraft, which can be found at University Airport, Davis, California, USA. The drag analysis performed aims at comparing the provided performance data of the aircraft from its pilot's operating manual. It was found out that the maximum speed of the aircraft calculated through the analysis is lower than the listed value. This was attributed to the conservative approximations made during the calculation of drag coefficients that modeled the airplane components as simple shapes. For its rate of the climb, which is another important performance parameter, it was seen that the calculated rate of climb was double of the listed value, indicating to discrepancies in calculating the maximum excess power of the aircraft during operation. Overall, it was seen that the set of assumptions made throughout the analysis significantly affected the results of the drag and performance analysis and should be carefully investigated for future performance evaluation of the aircraft.

TABLE OF CONTENTS

Nomenclature

Symbol	Meaning
C_D	Drag coefficient
C_{D_0}	Zero lift drag coefficient
C_L	Lift coefficient
AR	Aspect ratio
e	Oswald’s efficiency factor
$D_{parasitic}$	Parasitic drag
q_∞	Dynamic pressure
S	Reference area
f	Equivalent flat plate area
$C_{D_{0,w}}$	Wing zero lift drag coefficient
R_{wf}	Wing-fuselage interference factor
R_{LS}	Lifting surface correction factor
C_{f_w}	Wing turbulent flat plate coefficient
t/c	Thickness ratio of the wing at mean geo. chord
S_{wet_w}	Wetted area of the wing
K_w	Surface area factor
$S_{exposed_w}$	Exposed planform area of the wing
C_{f_f}	Fuselage turbulent flat plate coefficient
l_f/d_f	Fuselage fineness ratio
S_{wet_f}	Wetted area of the fuselage
C_{D_i}	Induced drag coefficient
P_{req}	Power required
P_{av}	Power available
V_∞	Freestream velocity
D	Drag force
η_p	Propeller efficiency

BHP	Brake horsepower
RC_{max}	Maximum rate of climb
μ_∞	Air dynamic viscosity
ρ_∞	Air density
Re	Reynolds number
C_{L_w}	Lift coefficient of wing
C_{L_f}	Lift coefficient of fuselage
Ma	Mach number
Λ	Sweep angle
$\bar{c}_{w_e}$	Mean geometric chord of the wetted wing

Introduction

One of the important parameters in aircraft design and performance evaluation is drag buildup and analysis. The drag of an aircraft essentially determines the performance characteristics of the plane, such as the maximum speed, maximum rate of climb, power required for different flight stages, and many more. In this case of drag analysis, a 1978 Cessna Model 152 is evaluated for its performance values by taking the information in the operating handbook and physical measurements taken at University Airport (KEDU) at Davis, California, USA. The three-view drawing of the aircraft taken from the Pilot's Operating Handbook is below:

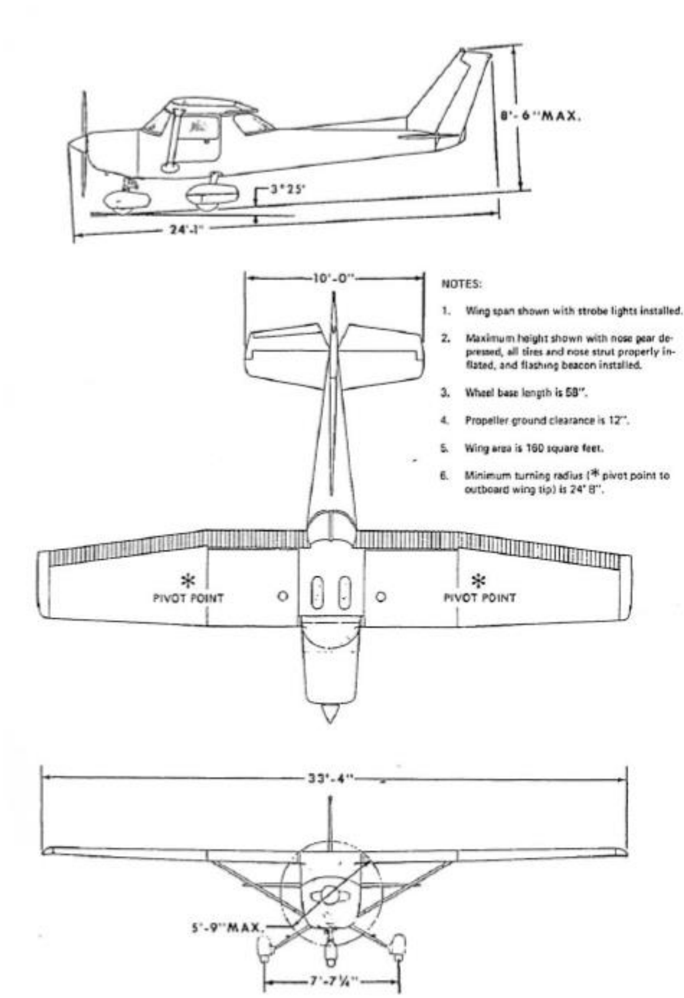

Figure 1-1. Three View

Figure 1. Three-View of 1978 Cessna Model 152. *From: [1] Pilot's Operating Handbook Cessna 152, 2nd ed., Cessna Aircraft Company, Wichita, KS, USA, 1977, Ch. 1-2.*

Cessna Model 152 is a two-seater aircraft with an Avco Lycoming Engine rated at 110 BHP for 2550 RPM. The airplane is fitted with a propeller, and for the drag analysis, it is assumed to have a constant propeller efficiency, and therefore a constant available power value. The general dimensions of the aircraft is given in the three-view in Figure 1. For the parasitic drag calculation, a field trip was made to University Airport to obtain the dimensions of the external aircraft components that contribute to the drag of the airplane. The aircraft analyzed is identified as 'N65415" and can be seen below:

Figure 2. Cessna 152 Analyzed at University Airport (KEDU), Davis, California, USA.

The airplane is rated for 1670 lbs for maximum take-off weight, maximum cruise speed of 107 knots (75% power at 8000 ft), and rate of climb of 715 ft/min. In this analysis, the listed vertical and cruise speeds are compared to the results from the drag analysis.

Methodology

For the drag buildup of the Cessna 152, the main drag components considered is as follows (van Dam, 4):

$$C_D = C_{D_0} + \frac{C_L^2}{\pi * AR * e} \qquad [2]$$

Where C_{D_0} is the combined drag coefficient of the individual aircraft components that contribute to drag of the airplane, such as the landing gears, pitot static tube, wing, fuselage, etc. while the second component is the induced drag due to lift. The second term can be calculated using cruise conditions where L = W, which helps obtain C_L.

To calculate the individual drag contributions of the external aircraft components, equivalent flat plate area method can be used where parasitic drag is:

$$D_{parasitic} = q_\infty * \sum_{j=1}^{n} C_{D_j} S_j = q_\infty * f = equivalent\ flat\ plate\ area$$

Where S_j is the reference area and $C_{D,j}$ is the drag coefficient of the part. Certain components of the aircraft require detailed calculation of the flat plate area, which includes the wing, fuselage, and the empennage.

For the wing, the zero-lift drag coefficient, $C_{D_0,w}$, is (Roskam, 148):

$$C_{D_0,w} = R_{wf} R_{LS} C_{f_w} \{1 + L'(t/c) + 100(t/c)^4\} \frac{S_{wet_w}}{S} \qquad [3]$$

R_{wf} is the wing-fuselage interference factor and can be obtained from Figure 5.11 in Airplane Aerodynamics and Performance by Roskam. R_{LS} is the lifting surface correction factor that is dependent on the sweep angle of the aircraft and can be obtained from Figure 5.12. C_{fw} is the turbulent flat plate friction coefficient of the wing and can be obtained from Figure 5.13 after calculating the wing Reynolds number, Re_{Nw}. L' is the airfoil thickness parameter from Figure 5.15, (t/c) is the thickness ratio of the wing at mean geometric chord. S_{wet_w} is the wetted area of the wing and S is the wing reference area:

$$S_{wet_w} = K_w S_{exposed_w}$$

$$K_w = 1.9767 + 0.5333\left(\frac{t}{c}\right)\ for\ t \geq 0.05$$

Same steps can be taken to determine the zero-lift drag coefficient for the horizontal and the vertical tail.

For the fuselage, the zero-lift coefficient can be found by another formula:

$$C_{D_{o_f}} = R_{wf} C_{f_f} \left\{1 + \frac{60}{\left(\frac{l_f}{d_f}\right)^3} + 0.0025\left(\frac{l_f}{d_f}\right)\right\} \frac{S_{wet_f}}{S}$$

In this formula, C_{f_f} is the turbulent flat plate friction coefficient from Figure 5.13, $\left(\frac{l_f}{d_f}\right)$ is the fuselage fineness ratio from Table 5.1, and S_{wet_f} is the wetted area of the fuselage. To use Table 5.1, the values for Cessna 185 is used as a substitute for 152, due to lack of data for the latter. Using engineering judgement, the fuselage of the 185 is determined to be most similar to the 152.

After adding up the equivalent flat plate areas of all the components, C_{D_0} can be found by dividing the f value by the total wetted area of the aircraft. Once all the C_{D_0} values are combined, drag force can be found by combining C_{D_0} and C_{D_i} along with the dynamic pressure and aircraft reference area.

After calculating the total drag force, the power required for various aircraft speed can be found using:

$$P_{req} = DV_{\infty}$$

In addition to the power required, the power available can be calculated using the given performance parameters of Cessna 152:

$$P_{av} = \eta_p BHP$$

These two power parameters can be plotted against aircraft speed, which yields the desired parameters to be investigated in this report: maximum rate of climb and maximum speed. Maximum speed is determined by the intersection of the two curves. Maximum rate of climb is determined by the following equation:

$$RC_{max} = \frac{Maximum\ Excess\ Power}{W} = \frac{\left(P_{av} - P_{req}\right)_{max}}{W}$$

Using this methodology, a detailed drag analysis can be performed on the selected aircraft.

Results

For the analysis, some initial parameters were set to easily calculate the drag coefficients of the individual aircraft components:

Table 1. [4] 1978 Cessna Model 152 and Freestream Parameters.

1978 Cessna Model 152 and Freestream Parameters	
Airspeed at Cruise: 107 kts = 180.6 ft/s	**Air Dynamic Viscosity at SL:** $3.737 * 10^{-7}$ lb*sec/ft^2
Air Density at SL: $2.377 * 10^{-3}$ slugs/ft^3	**Re/c:** $1.149*10^6$

The summary of the individual drag contributions of the aircraft external components is given in Table 2 below. The detailed calculations are included in Appendix A.

Table 2. 1978 Cessna Model 152 Aircraft Component Drag Contributions.

Component	Area [ft^2]	$C_{D,0}$	f = C_D * S [ft^2]	Reference
Wings	326.51	0.0088	1.4121	Roskam, J.
Fuselage	176.00	0.0098	1.7255	Roskam, J.
Nose Cone	0.7162	0.4672	0.3346	White, Frank M.
Nose Wheel	0.4219	0.6267	0.2588	White, Frank M.
Step-up Handle (x2)	0.0035	0.7300	0.0025	White, Frank M.
Exhaust	0.0174	0.6800	0.0118	White, Frank M.
Nose Landing Gear Cylinder	0.0208	0.7217	0.0150	White, Frank M.
Nose Landing Gear Wheel Holder	0.0069	1.1950	0.0083	White, Frank M.
Wing Strut (x2)	1.9722	0.1225	0.2416	White, Frank M.
Step (x2)	0.0039	1.4600	0.0057	White, Frank M.
Landing Gear Strut (x2)	0.6667	0.1175	0.0783	White, Frank M.
Landing Gear Wheel (x2)	1.2917	0.6155	0.2588	White, Frank M.
Pitot Tube	0.0137	0.7300	0.0100	White, Frank M.

Landing Strut Step (x2)	0.0104	1.2067	0.0126	White, Frank M.
Fuel Sump	0.0069	0.7640	0.0053	White, Frank M.
Horizontal Tail	61.22	0.0097	0.2919	Roskam, J.
Static Port	0.0052	1.1900	0.0062	White, Frank M.
Door Lock (x2)	0.0278	1.1850	0.0329	White, Frank M.
Vertical Tail	30.00	0.0097	0.1427	Roskam, J.
Beacon	0.0347	0.6400	0.0222	White, Frank M.
Wing Lights (x2)	0.0208	0.6533	0.0136	White, Frank M.
ADS-B	0.0347	0.6120	0.0213	White, Frank M.
Subtotal			f_1 = 4.9117	
Cooling drag			10% of f_1 = 0.4912	van Dam, C.P.
Interference drag			10% of f_1 = 0.5403	
Total			f_{tot} = 5.9432	

Using this chart, it is seen that the total flat plate area of the aircraft is 5.9432. Combined with the total wet area of the aircraft, C_{D_0} is found by:

$$C_{D_0} = \frac{f}{S_{wet,total}} = \frac{5.9432\ ft^2}{599.01 ft^2} = 0.0099217$$

Then, induced drag can be calculated by equating lift to the weight of the aircraft, which is 1670 lbs max. First, the lift coefficient for the wing and fuselage is calculated:

$$C_{L_w} = \frac{1670}{\frac{1}{2}(2.377 * 10^{-3})\left(\frac{180.6ft}{s}\right)^2 * 160} = 0.2693$$

$$C_{L_f} = \frac{1670}{\frac{1}{2}(2.377 * 10^{-3})\left(\frac{180.6ft}{s}\right)^2 * 176} = 0.2448$$

$$C_L = \left(C_{L_w} S_w + C_{L_f} S_f\right)\frac{1}{(S_{total})} = 0.2564$$

Then, the induced drag can be calculated using the following:

$$C_{D_i} = \frac{C_L^2}{\pi * AR * e} = \frac{0.2564^2}{\pi\left(\frac{33.33^2}{160}\right)(0.7)} = 0.004308$$

In this calculation, the Oswald's efficiency factor is assumed to be e = 0.7, as it is in the drag analysis notes (van Dam, 6).

Now, the total drag coefficient can be calculated by adding C_{D_i} and C_{D_0}:

$$C_D = C_{D_i} + C_{D_0} = 0.004308 + 0.0099217 = 0.0142285$$

As drag is a function of airspeed, the power required, P_{req}, can be found as a function of V_∞:

$$P_{req} = DV_\infty = C_D * \frac{1}{2} * \rho * V_\infty^3 * S$$

$$= (1.356/(7.457 * 10^2)(0.0142285)\left(\frac{1}{2}\right)(2.377 * 10^{-3})(599.01)V_\infty^3$$

In this formula, the highlighted part is the conversion factor from $slugs * ft^2/s^3$ to hp (Torenbeek, 515).

In addition, the power available for the aircraft is dependent on the engine and the propeller. For this analysis, the propeller efficiency, η_p, is estimated to be 75%. Then,

$$P_{av} = \eta_p BHP = (0.75)(110) = 82.5\ hp$$

Using these two power values, a power curve can be plotted:

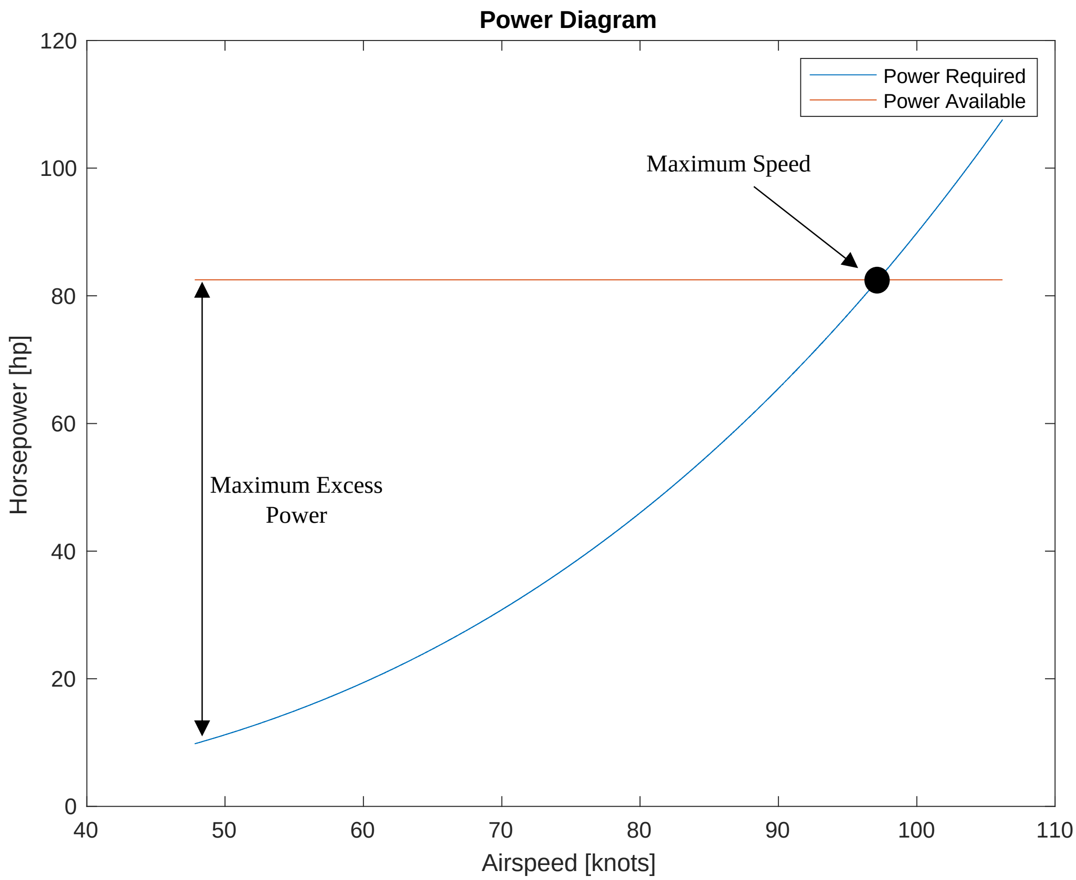

Figure 3. Calculated Power Curve of Cessna 152.

Looking at Figure 3, it can be seen that the maximum speed of the aircraft is around 97 knots. Also, the maximum excess power is the maximum difference between these two curves, which is 72.69 hp. So, the maximum rate of climb is:

$$RC_{max} = \frac{Maximum\ Excess\ Power}{W} = \frac{550 * 72.69}{1670} = 23.94\frac{ft}{s} = 1436\ ft/min$$

These values will be further discussed in the next section.

Discussion

In the project, the drag analysis was performed on a Cessna 152 in order to compare the performance values supplied by the manufacturer to the estimated performance values. Observing the Pilot's Operating Manual, it is stated that the maximum speed of the aircraft is 110 knots while the rate of climb is rated as 715 ft/min.

As drag force on an aircraft ultimately determines the limits of the aircraft in terms of vertical and horizontal speed where the power available from the propulsion systems set the limit on the power that can be used to reach to a higher speed. Therefore, a detailed drag analysis with consideration of all the significant external aircraft components is key to evaluating the performance statistics of an airplane. This project is also important for the future RFP work that requires evaluating the team's own design of a short range high capacity transport aircraft.

The results found from the drag analysis show that the maximum speed of Cessna 152 is 97 knots with a maximum rate of climb of 1436 ft/min. The speed estimated from drag analysis shows a lower value for maximum speed, which is 13% slower. This result makes sense in the way where the drag analysis performed was conservative as a result of the assumptions made during the drag coefficient and equivalent flat plate area calculations. The areas used for the drag calculations simplified the protrusions on the aircraft as simple shapes, such as cylinders, flat plates, and 3D ellipses, instead of streamlined shapes that significantly reduce the drag. Because measuring the exact wetted area of these components were not viable by the means of using a measuring tape, a caliper, and a ruler, it was decided that the use of simple shapes would help the processing time of the raw data significantly. In addition, the use of the simple shapes enabled the author to reference to the textbook, Fluid Mechanics by Frank M. White, which includes drag coefficient charts for certain defined shapes. In addition, the fuselage areas, wetted and reference, were not included in Table 5.1 in *Airplane Aerodynamics and Performance,* by Jan Roskam. Therefore, the areas were used were in fact of a Cessna 185, which was considered to be the most similar airplane structure to a Cessna 152. For future work, these assumptions can be replaced by more precise measurements and modeling of the aircraft that would yield more accurate results, bringing the overall drag of the airplane down, and increasing the maximum speed of the aircraft.

The rate of climb found through the analysis is 1436 ft/min, which is 100% higher than the rated rate of climb. This value is significantly higher, therefore indicates to an error in calculation of the maximum excess power. While calculating the induced drag of the aircraft, the induced drag coefficient was found for cruise parameters, including the speed of the aircraft. In a more accurate calculation, the induced drag would depend on the speed of the aircraft, which would yield a different power available curve. If the current model for calculations is substituted by a varying induced drag coefficient, it is expected that the maximum excess power would occur somewhere in between the stall velocity and the maximum velocity of the aircraft. This way, the rate of climb can be calculated more accurately and account for variations in freestream parameters in the power curve functions.

Conclusions

The drag analysis and performance characteristics of the Cessna 152, which was the main focus of the project, showed the effect of assumptions made during the modeling of the aircraft. The key ideas used in the analysis included utilizing the power curves for an aircraft, which indicate the maximum available power and the required power for various speeds. From the two curves plotted, it was seen that the maximum speed of the aircraft was 97 knots while the operating manual indicated 110 knots. These two values were considered close and the discrepancy was attributed to the shape approximation of the external aircraft components in order to calculate the drag coefficients.

On the other hand, the rate of climb of the aircraft was found via calculating the maximum excess power of the aircraft and dividing the value by the maximum weight of the aircraft. The value found was 1436 ft/min, which was significantly different than the listed value in Pilot's Operating Manual, which was 715 ft/min. The difference between these two values indicated a major source of error in the calculation of the minimum power required for equilibrium flight across various values of airspeed. The suspected error is due to the calculation of lift-induced drag, which was calculated to be a single value assuming cruise conditions, while it should have been a function of airspeed. For future drag analyses for this aircraft, it is recommended that the aircraft is modeled in a more detailed fashion where the actual shapes of the aircraft is incorporated for drag coefficient calculations. One possible method would include using CFD in order to capture a more accurate drag force estimation.

References

[1] [4] *Pilot's Operating Handbook Cessna 152*. 2nd ed. Cessna Aircraft Company. Wichita, KS, USA. 1977. Ch. 1-2.

[2] van Dam, C.P. *EAE 130A – Aircraft Performance & Design - Aircraft Drag Buildup and Analysis.* 20 January 2020.

[3] Roskam, J. and Lan, C.T. *Airplane Aerodynamics and Performance*. DARcorporation. 1997.

[5] White, Frank M. *Fluid Mechanics*. McGraw-Hill Education, 2016.

Torenbeek, Egbert, and H. Wittenberg. *Flight Physics: Essentials of Aeronautical Disciplines and Technology, with Historical Notes*. Springer, 2009.

Appendices

Appendix A1. Example Aircraft Component Drag Calculations – Wings

$$C_{D_0,w} = R_{wf} R_{LS} C_{f_w} \{1 + L'(t/c) + 100(t/c)^4\} \frac{S_{wet_w}}{S}$$

$$Re_{Nf} = \frac{\rho V_\infty l_f}{\mu} = 2.77 * 10^7$$

$$Ma = \frac{a}{V} = 0.16$$

Mach number used for the charts is Ma = 0.25, due to lack of data for Ma<0.25.

$$R_{wf} = 1.06\ (From\ Figure\ 5.11)$$

$$cos\Lambda = 1 \rightarrow R_{LS} = 1.08 (From\ Figure\ 5.12)$$

$$S_{exposed_w} = 160 ft^2$$

$$S_{wet_W} = K_w S_{exposed_W}$$

$$K_w = 1.9767 + 0.5333 \left(\frac{t}{c}\right) = 2.04\ for\ t \geq 0.05$$

$$S_{wet_W} = K_w S_{exposed_W} = (2.04)(160) = 326.51 ft^2$$

$$\bar{c}_{w_e} = \frac{wing\ area}{wingspan} = \left(\frac{160}{29.88}\right) = 5.36 ft$$

$$Re_{Nw} = \frac{\rho V_\infty \bar{c}_{w_e}}{\mu} = 6.15 * 10^6$$

$$C_{f_w} = \frac{0.455}{(\log_{10} R_N)^{2.58}(1 + 0.144 Ma^2)^{0.58}} = 3.24 * 10^{-3}\ (From\ Figure\ 5.13)$$

NACA 2412 Wings → $\left(\frac{t}{c}\right)$ max @ $0.3c \rightarrow L' = 1.2\ (From\ Figure\ 5.15)$

$$\left(\frac{t}{c}\right) = 0.12$$

Using all these coefficients,

$$f_{wing} = C_{D_0,w}S = 1.4121$$

Wing drag due to compressibility is negligible due to a low Mach Number.

Same calculations can be applied to the vertical and the horizontal wing by changing the airfoil used in the equations to NACA 0012.

Appendix A2. Example Aircraft Component Drag Calculations – Fuselage

$$C_{D_{o_f}} = R_{wf}C_{f_f}\left\{1 + \frac{60}{\left(\frac{l_f}{d_f}\right)^3} + 0.0025\left(\frac{l_f}{d_f}\right)\right\}\frac{S_{wet_f}}{S}$$

$$R_{wf} = 1.06\ (From\ Figure\ 5.11, same\ as\ the\ wing)$$

$$Re_{Nf} = \frac{\rho V_\infty l_f}{\mu} = 2.77 * 10^7$$

$$C_{f_f} = \frac{0.455}{(\log_{10} R_N)^{2.58}(1 + 0.144Ma^2)^{0.58}} = 2.56 * 10^{-3}\ (From\ Figure\ 5.13)$$

To use Table 5.1, the values for Cessna 185 is used as a substitute for 152, due to lack of data for the latter. Using engineering judgement, the fuselage of the 185 is determined to be most similar to the 152. Therefore,

$$\frac{l_f}{d_f} = 5.15$$

$$S = 176\ ft^2$$

$$S_{wet_f} = 292 ft^2$$

Using the found coefficients,

$$C_{D_{o_f}} = 0.0065$$

50% is added to the drag coefficient of the fuselage to account for the canopy, as it was done in the Drag Analysis Notes (van Dam, 6)

Then, the drag coefficient is:

$$C_{D_{o_f}} = 0.0098$$

Appendix A3. Example Aircraft Component Drag Calculations – Nose Cone

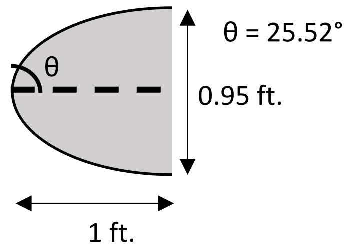

Figure 4. Nose Cone Drawing of Cessna 152.

Referencing to Frank M. White's Fluid Mechanics 8th Edition Textbook, Table 7.3, pg. 483:

Body	C_D based on frontal area							
Cone: [60]	θ:	10°	20°	30°	40°	60°	75°	90°
	C_D:	0.30	0.40	0.55	0.65	0.80	1.05	1.15

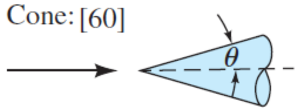

Figure 5. Drag Coefficient Chart for a Cone. [5]

The Reynolds number is calculated to check if the CD can be approximated from this graph, which states that, to use this table $Re \geq 10^4$. Using the *Re/c* parameter in Table 1, Reynolds number is calculated to be Re = 1.097 * 10^6. Drag coefficient for the nose cone of Cessna 152 is determined to be between 0.40 and 0.55, which can be interpolated and found to be C_D = 0.4672.

For the other components, same method is applied by looking up the C_D charts of various shapes that the components were approximated.