EAE 129 Final Report:

Simulation and Analysis of Aircraft Response Characteristics

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Abstract

Dynamic stability is an important parameter for defining the overall stability of an aircraft. In this experiment, a prototype of the Learjet C-21 was tested at sea level to analyze the longitudinal dynamic stability of the aircraft. The data obtained was used to evaluate certain dynamic stability characteristics such as model characteristics, time responses, reduced order analysis, and handling qualities investigation. After evaluation, these parameters proved this aircraft to be dynamically stable where the unit step or unit impulse input applied to the aircraft results in a dampened oscillation response of the aircraft. These results indicate that the transient motion of the aircraft leads to a stable steady state output.

Introduction

Dynamic stability is an important aspect of the aircraft design process as it is essential for the control and maneuverability of the aircraft. Aircraft dynamic stability is the transient motion that is included in the process of recovering from perturbed conditions. The system is said to be dynamically stable when the oscillation of aircraft motion dampens out over time, where the system is dynamically unstable when the oscillation gets worse over time. Another case is that the aircraft has neutral dynamic stability when the aircraft has oscillations that never dampen out.

The prerequisite for dynamic stability is the static stability of the system. Static stability determines the aircraft response to the perturbations to the trimmed condition of the aircraft, which the aircraft is said to have positive static stability when the aircraft returns to the trimmed condition, while it is said to have negative static stability when the aircraft diverges from its initial equilibrium position. Although all dynamically stable systems are statically stable, not all statically stable systems are dynamically stable. Therefore, in this experiment, the Learjet C-21 aircraft is assumed to be statically stable in order to analyze its dynamic stability. As there are different types of static stability including longitudinal, lateral, and directional static stability, the same principle applies to dynamic stability.

In order to understand the procedures undertaken, some mathematical concepts might be useful in order to perform the dynamic stability analysis. As the aircraft response can be modeled as a second order system, the important parameters are the time constant, natural frequency, damping ratio, and damped frequency. Time constant (τ) is defined as the lag associated with the observed rise to the steady-state value, as it is important for evaluating the aircraft response properties. When the time constant is smaller, it means that the aircraft responds faster to the input provided. The natural frequency (ω_n) represents the system frequency which it would oscillate in the absence of damping. It should be noted that this is not the frequency which the system oscillates, but it is the highest frequency that is capable of oscillating. The actual oscillation frequency of the system is called the damped frequency (ω_d). The damping ratio of the system (ξ) is the indication of the system's stability. This value is between -1 and 1 for underdamped systems whereas it is between 0 and 1 for stable systems. These values can be used to represent the roots of the time response of the aircraft where:

$$P = a + bi \, , a = -\xi\omega_N \, , b = \omega_N\sqrt{1-\xi^2}$$

The roots, a and b, can be represented on the real and imaginary plane which one can see the damping and oscillation of the time response:

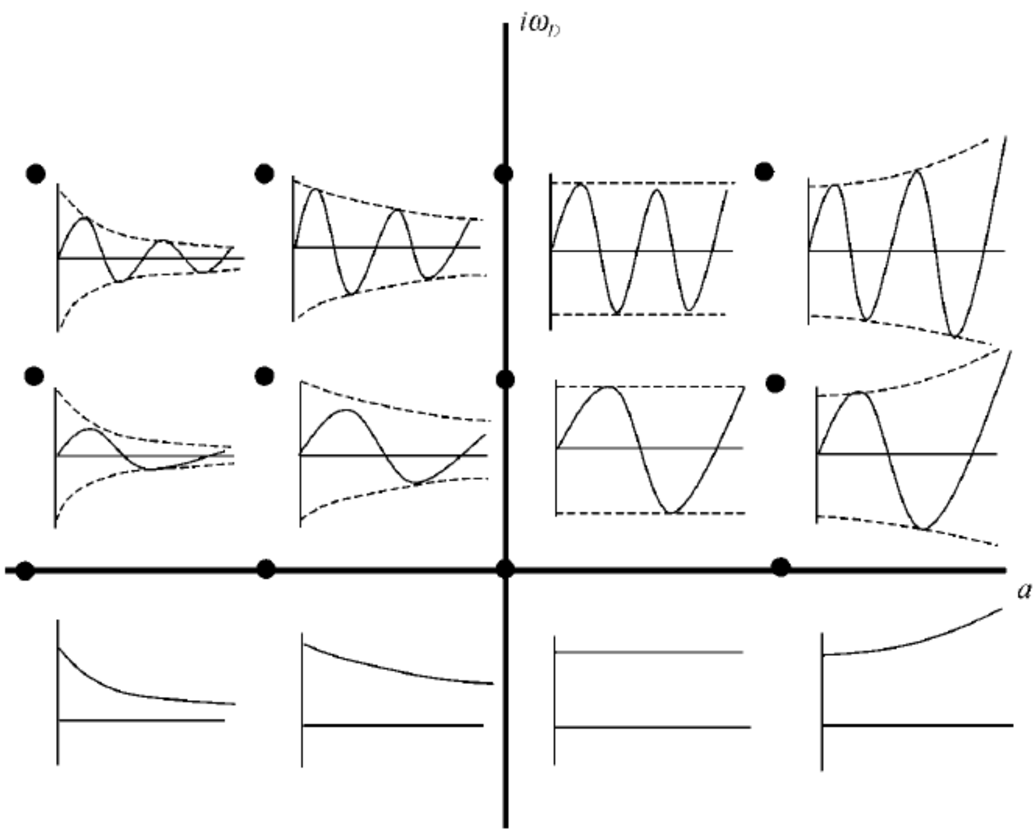

Figure 1. *Time response of a second-order system for different roots.[1]*

This chart is useful to determine the dynamic stability and damped frequency of the system as a negative a value indicates a dynamically stable system whereas a positive a value means that the system is unstable. When a is equal to zero, the system oscillates indefinitely.

Transfer functions are used to determine the aircraft response by implementing Laplace transform in the process. Transfer function is the ratio of the Laplace transformation of the output to the Laplace transformation of the input. The input and output can be any aircraft response parameter such as roll angle response, aileron deflection angle, etc. An important parameter obtained from the transfer function is the characteristic equation, which is the denominator of the transfer function. The roots of this characteristic equation define the dynamic characteristic of the system such as the time constant (first-order), damping ratio and the natural frequency of the (second-order) aircraft response.

To analyze the longitudinal dynamic stability, the longitudinal linearized equations of motion in Laplace form are used. These equations are converted into transfer functions where one can find the characteristic equations. Different transfer functions in this set have the same characteristic equation for the dynamic characteristics of the aircraft. For longitudinal dynamic stability, the aircraft is said to have two dynamic modes which are called short period mode and phugoid mode. These are different characteristic equations that show the respective dynamic characteristics of the mode. These two modes vary where the short period mode has a greater damping ratio and natural frequency than the phugoid mode.

The longitudinal linearized equations of motion can be approximated to two degrees of freedom where the time response of u is almost constant. This allows the equations to be reduced where it is easier to find the roots of the characteristic equation, which the process is called Reduced Order Analysis.

After obtaining the parameters required to analyze the dynamic stability of the aircraft, one can assess the handling qualities of the aircraft. This can be done using the dynamic stability guidelines that are set as a military specification called MIL-F-8785C. This specification is no longer required for military aircraft, but it is a good indication of an aircraft's handling and flying qualities. These guidelines include the aircraft class, flight category, and the flying quality levels, which the MIL-F-8785C Flying Quality Levels can be seen below:

Level 1	Flying qualities clearly adequate for the mission flight phase
Level 2	Flying qualities adequate to accomplish the mission flight phase, but some increase in pilot workload or degradation in mission effectiveness, or both, exists
Level 3	Flying qualities such that the airplane can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate, or both. Category A flight phases can be terminated safely, and Category B and C flight phases can be completed

Figure 2. *MIL-F-8785C Flying Quality Levels.[1]*

Results

After the initial test of the prototype at the sea level, following parameters were measured:

Table 1. *Aircraft parameters and stability derivatives.*

Parameter (unit)	Value	Parameter (unit)	Value
Airspeed V_∞ (ft/s)	170	Dynamic pressure $\bar{q}$ (lb/ft^2)	34.3
Weight W (lb)	13,000	Wing area S (ft^2)	230
Wing span b (ft)	34.0	Wing chord $\bar{c}$ (ft)	7.0
c.g. ($\bar{c}$)	0.32	Trim AOA α_{trim} (deg)	5.0
I_{xx} (slug-ft^2)	2.79*10^4	I_{yy} (slug-ft^2)	1.88*10^4
I_{zz} (slug-ft^2)	4.11*10^4	I_{xz} (slug-ft^2)	-3.60*10^2
X_u (1/s)	-0.0589	X_α (ft/s^2)	11.3335
Z_u (1/s)	-0.3816	Z_α (ft/s^2)	-103.4862
Z_{δ_e} (ft/s^2)	-7.8162	M_α (1/s^2)	-1.9387
$M_{\dot{\alpha}}$ (1/s)	-0.3024	M_q (1/s)	-0.8164
M_{δ_e} (1/s^2)	-2.8786	M_u (1/ft.s)	-0.0002

Using this data, longitudinal linearized equations of motion can be written in the following form:

$$\begin{bmatrix} (s - X_u - X_{T_u}) & -X_\alpha & g\cos\Theta_1 \\ -Z_u & [s(U_1 - Z_{\dot{\alpha}}) - Z_\alpha] & [-(Z_q + U_1)s + g\sin\Theta_1] \\ -(M_u + M_{T_u}) & -[M_{\dot{\alpha}}s + M_\alpha + M_{T_\alpha}] & (s^2 - M_q s) \end{bmatrix} \begin{bmatrix} \dfrac{u(s)}{\delta_e(s)} \\ \dfrac{\alpha(s)}{\delta_e(s)} \\ \dfrac{\theta(s)}{\delta_e(s)} \end{bmatrix}$$

$$= \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \tag{7.27}$$

Figure 3. *Longitudinal linearized EOM in matrix form.[1]*

where $u(s)/\delta_e(s)$, $\alpha(s)/\delta_e(s)$, $\theta(s)/\delta_e(s)$, are the transfer functions of the EOMs. This set of equations can be solved using a computer program such as MATLAB. The solutions are as follows:

$u(s)/\delta_e(s)$:

```
tfu_zpk =

            -0.52109 (s-114.5) (s+0.8977)
  ---------------------------------------------
  (s^2 + 0.03449s + 0.05567) (s^2 + 1.752s + 2.447)

Continuous-time zero/pole/gain model.
```

$\alpha(s)/ \delta_e(s)$:

```
tfa_zpk =

   -0.045978 (s+63.43) (s^2 + 0.05778s + 0.07125)
  ---------------------------------------------
  (s^2 + 0.03449s + 0.05567) (s^2 + 1.752s + 2.447)

Continuous-time zero/pole/gain model.
```

$\theta(s)/ \delta_e(s)$:

```
tft_zpk =

            -2.8647 (s+0.5259) (s+0.1136)
  ---------------------------------------------
  (s^2 + 0.03449s + 0.05567) (s^2 + 1.752s + 2.447)

Continuous-time zero/pole/gain model.
```

These transfer function now can be used to analyze the dynamic stability of the aircraft, including model characteristics, time responses, reduced order analysis, and handling qualities investigation.

Discussion

Model Characteristics

All three transfer functions found in the results indicate the same characteristic equations, which are:

$$C_1 = (s^2 + 0.03449s + 0.05567) \text{ and } C_2 = (s^2 + 1.752s + 2.447)$$

From the characteristic equations, one can determine the different natural frequencies and damping ratios of the system:

$$\omega_{N_1} = 0.2359$$

$$\xi_1 = 0.0731$$

$$\omega_{N_2} = 1.5642$$

$$\xi_2 = 0.5600$$

Also, the roots of these equations provide useful information about the dynamic modes of the aircraft, which are calculated:

$$r_1 = -0.0172 \pm 0.2353i$$

$$r_2 = -0.8760 \pm 1.2959i$$

The roots are in the real-imaginary plane where both real and imaginary parts are nonzero. The real part is negative meaning that this system appears to be dynamically stable. In addition, the nonzero imaginary part indicates that the aircraft response has oscillations.

The aircraft response has two longitudinal dynamic modes as it can be seen by the two characteristic equations the transfer function has. Each characteristic equation points out to a different mode that were discussed earlier, short period mode and phugoid mode. The short period mode has a higher natural frequency so the characteristic equation 1 (C_1) can be identified as the short period mode while the characteristic equation 2 (C_2) can be identified as the phugoid mode. So, the natural frequency and damping ratio of the *short period mode* is as follows:

$$\omega_{N_{SP}} = 1.5642$$

$$\xi_{SP} = 0.5600$$

Similarly, the natural frequency and damping ratio of the *phugoid mode* is as follows:

$$\omega_{N_{PH}} = 0.2359$$

$$\xi_{PH} = 0.0731$$

Time Responses

The time responses for u, α, and θ can be plotted using the built-in MATLAB functions step() and impulse() and the previously obtained transfer functions. This way the aircraft response to unit impulse and unit step inputs can be seen.

The time response for u is as follows:

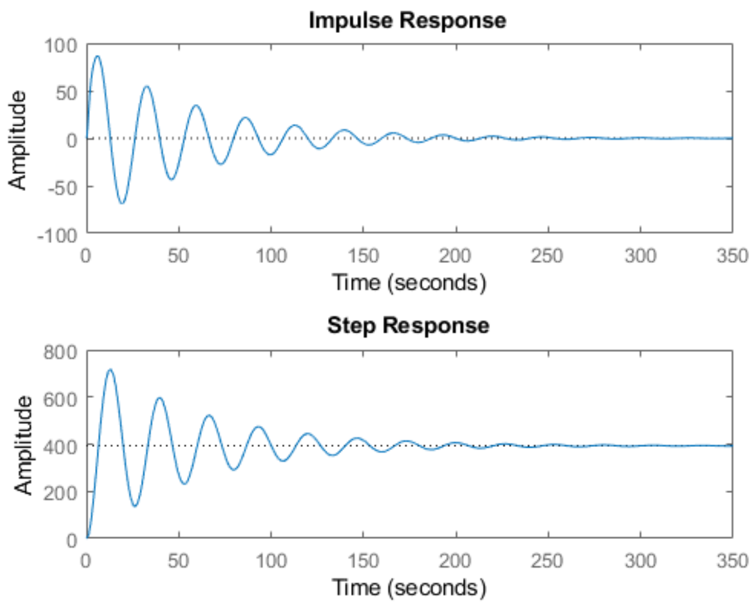

Figure 4. *Time response for u.*

The time response for α is as follows:

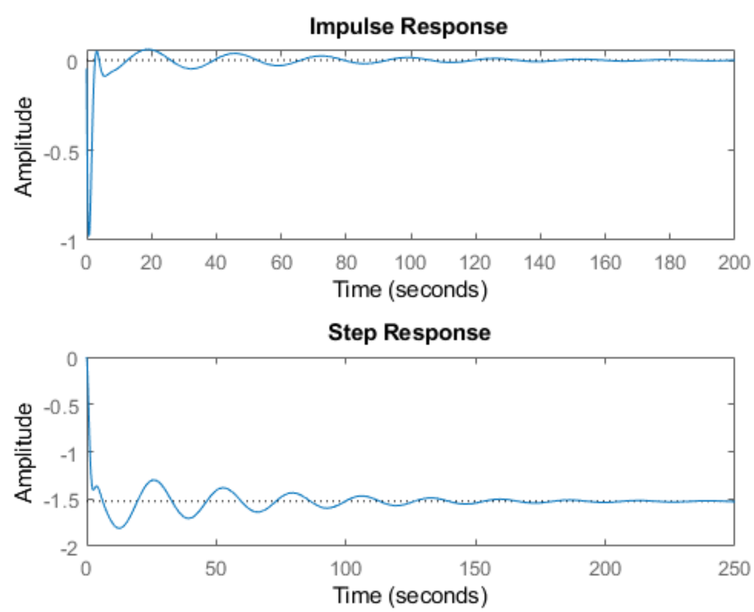

Figure 4. *Time response for α.*

The time response for θ is as follows:

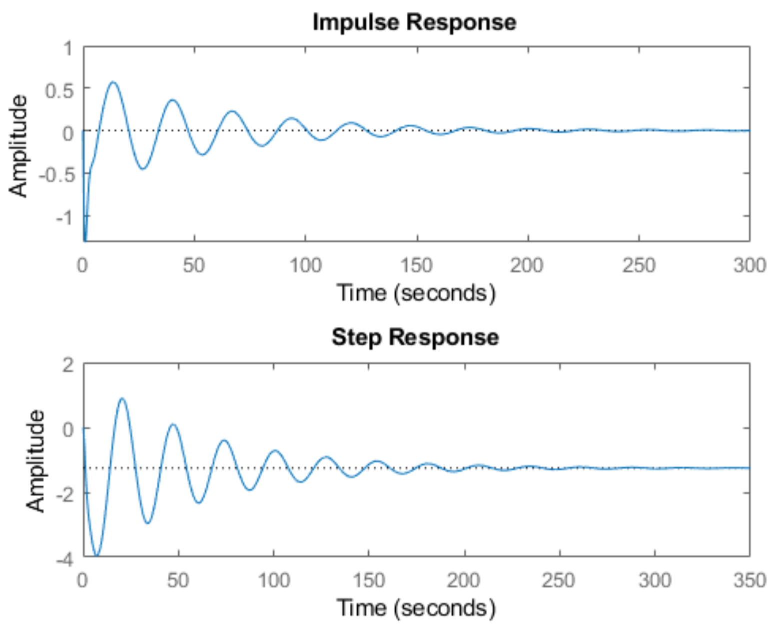

Figure 5. *Time response for θ.*

Looking at the six figures above, it appears that the aircraft response to unit step and unit impulse inputs is stable, as the oscillation dampens out over time.

<u>Reduced Order Analysis</u>

The longitudinal linearized EOMs can be reduced to two-degrees-of-freedom by approximating the short period response in u to be nearly constant. This way, the characteristic equation of the transfer function reduces to the following form:

$$s^2 - (M_q + \frac{Z_\alpha}{U_1} + M_{\dot{\alpha}})s + (\frac{Z_\alpha M_q}{U_1} - M_\alpha) = 0$$

This transfer function is calculated for the Learjet aircraft, which then found out to be the equation below:

$$C_{RO} = (s^2 + 1.728s + 2.436)$$

Then, the roots of the above characteristic equation for the reduced order analysis are found:

$$r_{RO} = -0.8638 \pm 1.2998i$$

In addition to the roots of the characteristic equation, one can also calculate the natural frequency and the damping ratio of the short period:

$$\omega_{N_{RO}} = 1.5607$$

$$\xi_{RO} = 0.5535$$

Looking at these values, it can be said that the reduced order approximation is accurate. Using the reduced order analysis, the error in the natural frequency of the short period is 0.22% while the error in the damping ratio of the short period is 1.16%. The time responses for unit impulse input can be modeled using the two different sets of natural frequency and damping ratio values:

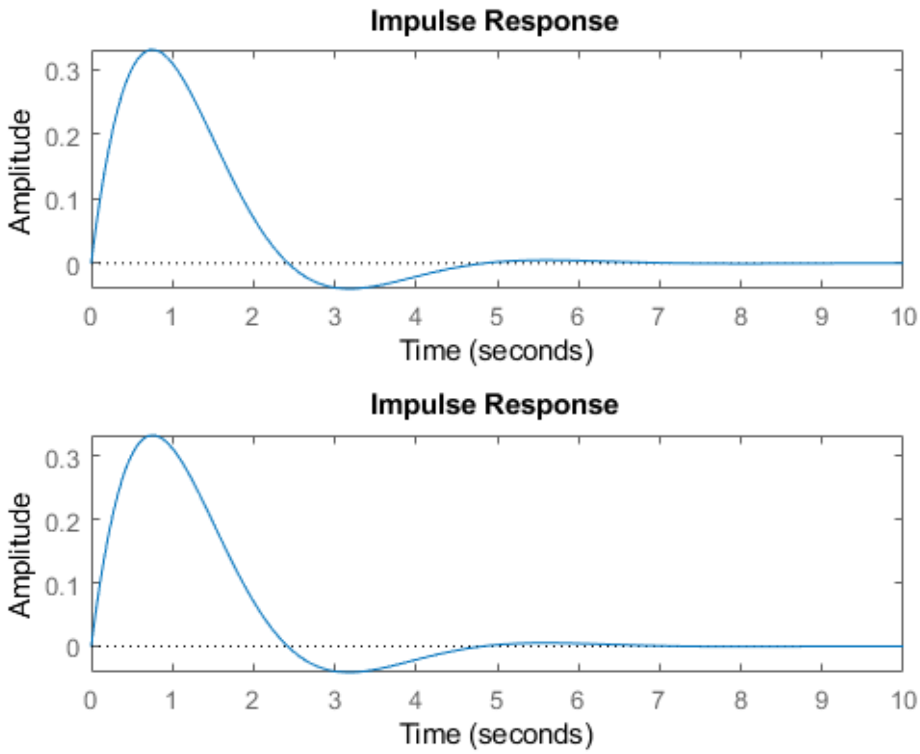

Figure 6. *Time response for unit impulse input for short period, using the standard analysis and the reduced order analysis.*

Handling Qualities Investigation

Based on the calculated values of the damping ratios for short period mode (ξ_{SP}) and phugoid mode (ξ_{PH}), the aircraft handling qualities can be assessed for different flight conditions. In this report, the flight condition Category B is evaluated.

Table 2. (a) *Short period damping ratio (ξ_{SP}) limits for Category B Flight Phases.* **(b)** *Phugoid damping ratio (ξ_{PH}) requirements.*

	Minimum	Maximum
Level 1	0.30	2.00
Level 2	0.20	2.00
Level 3	0.15	No maximum

(a)

	Requirement
Level 1	$\zeta_{PH} > 0.04$
Level 2	$\zeta_{PH} > 0$
Level 3	$T_2 > 55\text{s}$

(b)

Based on the tables above, the Learjet aircraft has a handling quality of *Level 1* as the short period damping ratio is between 0.30 and 2.00 and the phugoid damping ratio is greater than 0.04. The handling quality (Level 1) of the aircraft shows that the aircraft flying qualities is adequate for the mission flight phase.

The aircraft, Learjet C-21, was found out to be dynamically stable while having a handling quality of Level 1, meaning that the aircraft is suitable for a Category B Flight Phase. If the aircraft was found out to be dynamically unstable in one of its modes, short period mode or phugoid mode, one can improve the dynamic stability characteristics of the aircraft by redesigning some aspects of the aircraft.

$$\omega_{n_{sp}} \approx \sqrt{-M_\alpha} = \sqrt{\frac{-C_{m_\alpha}\bar{q}_1 S\bar{c}}{I_{yy}}} \qquad \zeta_{SP} \approx \frac{-\left(M_q + \frac{Z_\alpha}{U_1} + M_{\dot{\alpha}}\right)}{2\omega_{n_{SP}}}$$

Figure 7. *Short Period two-degrees-of-freedom approximation[1].*

Looking at these short period approximations, one can improve the dynamic stability by tweaking the natural frequency and the damping ratio accordingly, which can individually be adjusted by the stability derivatives and aircraft parameters that are related to these elements.

Conclusion

Main goal of the prototype testing of Learjet C-21 was to analyze the dynamic stability of the aircraft. The main focus was on the model characteristics, time responses, reduced order analysis, and the handling qualities of the aircraft. In order to calculate these parameters, the Learjet C-21 prototype was tested at sea level and data was obtained. The data obtained included the aircraft parameters and the stability derivatives.

Using the data, transfer functions were created in order to plot the time response of the aircraft for different types of longitudinal motion, as well as to obtain the natural frequency and the damping ratio of the system for different dynamic modes. The damping ratio of the system was between 0 and 1, which meant that the system was stable. In addition to that, the time response also showed that the aircraft response showed a dampening oscillation occurring after the perturbation of the aircraft. Overall, this aircraft can be considered to be longitudinally dynamically stable.

References

[1] Yechout, Thomas R., Morris Steven L., Bossert David E., Hallgren, Wayne F. *Introduction to Aircraft Flight Mechanics, Performance, Static Stability, Dynamic Stability, and Classical Feedback Control.* AIAA Education Series, 2003. p. 175.