Short Communication

Inducing compressive residual stress in microscale print-lines for flexible electronics

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Abstract

Printed electronics are becoming widespread in modern industrial devices. In many of the manufacturing processes of printed electronics, one step is the deposition of initially molten (or liquid), microscale, “print-lines” of material onto a flexible substrate. As the deposited molten print-line solidifies, the bonded print-line and substrate may have the tendency to curl (attain a finite curvature), due to the differences in the thermal expansion coefficients, elastic properties, etc. The quality and durability of the solidified print-line (which is mechanically-weak) is adversely affected by residual tensile stress states. Tensile stress states have a tendency to induce damage in the form of cavities or cracks in the deposited material, which would hinder the printed electronics operation. Therefore, ideally, one would like the solidified print-line to be in a state of compression. Inducing a compressive stress-state in the print-line is particularly important in the vicinity of the substrate interface, since damage in that location may also initiate delamination of the deposited material and, eventually, a malfunction of the intended printed electronics application. In this work, employing an elementary thermo-mechanical model, a mathematical expression is derived for the combination of system parameters needed to ensure that the print-line material at remains in a compressive state at the bimaterial interface.

1. Introduction

The motivation for this work is the large increase in the use of print-based manufacturing. In particular, printed electronics on flexible foundational substrates are becoming popular, and have a wide-range of applications, such as flexible solar cells and smart electronics. One important technological obstacle is to develop inexpensive, durable electronic material-units that reside on flexible platforms/substrates which can be easily deployed onto large surface areas. Print-based methods are, in theory, ideal for large-scale applications, and provide a framework for assembling electronic circuits by mounting printed electronic devices on flexible plastic substrates, such as polyimide and “PEEK” (Poly-Ether–Ether–Ketone, a flexible thermoplastic polymer) film. There are many variants of this type of technology, which is sometimes referred to as flexible electronics or flex circuits. Flex circuits can be, for example, screen printed silver circuits on polyester. For an early history of the printed electronics field (see Gamota, Brazis, Kalyanasundaram, & Zhang, 2004).

Regardless of the exact method of print-line deposition, the common physics is a curing bimaterial system of disparate properties. The objective of this paper is to provide a design criteria to achieve high-quality print-lines (Fig. 1). One important criteria for a high-quality print-line is that the initially molten (or liquid) deposited print-line material be in compression when the system cures/cools. The reasoning for this is that a compressive state is much more resistant to damage than a...
tensile state. Attaining a compressive stress-state in the print-line is critical in the vicinity of the bimaterial interface, since the damage in that location may also initiate delamination of the deposited material and, ultimately, a complete breakdown of the printed electronics application. In this paper, we focus on isolated print-line deposited on a strip (coupon) of substrate material. The overall idealized process that we analyze is as follows, using a simple one-dimensional stress-state (“strength-of-materials”) model for each strip:

- The “hot” material (print-line/substrate) are bonded together, and are initially of equal length.
- The materials are cooled by \( \Delta \theta = \theta - \theta_0 \).
- The interface remains perfectly bonded.
- The system is in static equilibrium.
- The temperature of the system is uniform.

Remark 1. In order to develop flexible micro/nanoelectronics for large area deployment, traditional methods of fabrication using silicon-based approaches have become limited for applications that involve large-area coverage, due to high cost of materials and equipment (which frequently need a vacuum environment). For flexibility and lower cost, the ability to develop these electronics on plastics is necessary. To accomplish this task, print-based technologies are starting to become popular for these applications. In many cases, this requires the development of nanoparticle-functionalized “inks”. These nanoparticles include germanium (which has higher mobility and better tailorable absorption spectrum for ambient light than silicon) and silver (which is being studied due to the possibility to sinter the particles even without the need of directly applied intense heating). Other semiconductor nanoparticles, including zinc and cadmium based compounds and metals, such as gold and copper, can be considered. Precise patterning of (nanoparticle-functionalized) prints is critical for a number of different applications. For example, some recent applications include low temperature electrode deposition (Huang, Liao, Molesa, Redinger, & Subramanian, 2003) optical coatings and photonics (Maier & Atwater, 2005; Nakanishi et al., 2009) biosensors (Alivisatos, 2004), catalysts (Haruta, 2002) and MEMS applications (Fuller, Wilhelm, & Jacobson, 2002; Ho et al., 2006). Several methods are available for patterning nanoparticles, the most popular of which is inkjet printing (Ridley, Nivi, & Jacobson, 1999; Sirringhaus et al., 2000). Inkjet printing is attractive due to its simplicity, high throughput, and low material loss. However, patterning with inkjet printing is limited to a resolution of around 20–50 \( \mu m \) with current printers (Ridley et al., 1999) with higher resolution possible by adding complexity to the substrate prior to printing (Wang, Zheng, Li, Huck, & Sirringhaus, 2004). Electrohydrodynamic printing has also been proposed to increase the resolution beyond the limits of inkjet printing, achieving a line resolution as small as 700 \( nm \) (Park et al., 2007). Due to the nature of the deposited materials, they are adversely affected by tensile stress states when they solidify.

2. An idealized purely extensional deformation

In order to initially illustrate some simple concepts, consider a simplified first problem, comprised of a system of two bonded layers, with no induced curvature (bending). We assume:

- The strips are bonded together, initially at the same temperature and all the same length.
- The strips are cooled down simultaneously.
- The strips have different stiffnesses and thermal expansion coefficients than the inner layer.
Referring to Fig. 2, there are two contributions to the strain. The strain due to the extensional load in material 1 is
\[ \varepsilon_1^e = \frac{F_1}{E_1A_1} \] (1)
and in material 2 is
\[ \varepsilon_2^e = \frac{F_2}{E_2A_2}. \] (2)

The thermal contribution in material 1 is
\[ \varepsilon_1^h = \alpha_1 \Delta \theta \] (3)
and in material 2 is
\[ \varepsilon_2^h = \alpha_2 \Delta \theta. \] (4)

For perfectly bonded one-dimensional strip kinematics, one equates the total normal strain in both materials to give
\[ \varepsilon_1^e + \varepsilon_1^h = \varepsilon_2^h + \varepsilon_2^e, \] (5)
which implies
\[ \frac{F_1}{E_1A_1} + \alpha_1 \Delta \theta = \frac{F_2}{E_2A_2} + \alpha_2 \Delta \theta. \] (6)

Since the system is in static equilibrium, we must also have that the sum of the forces is zero
\[ \sum F = F_1 + F_2 = 0. \] (7)

Solving Eqs. (6) and (7) simultaneously for the force in material 1 yields:
\[ F_1 = \frac{(\alpha_2 - \alpha_1) \Delta \theta}{K} \] (8)
and for the force in material 2:
\[ F_2 = \frac{(\alpha_2 - \alpha_1) \Delta \theta}{K}, \] (9)
where
\[ K \overset{\text{def}}{=} \frac{1}{E_1A_1} + \frac{1}{E_2A_2}. \] (10)

The total strain in material 1 is
\[ \varepsilon_1^{\text{tot}} = \frac{(\alpha_2 - \alpha_1) \Delta \theta}{E_1A_1K} + \alpha_1 \Delta \theta \] (11)
and for material 2 is

Fig. 2. An illustrative first problem: a system of bonded strips with an idealization of no thermally-induced curvature changes.
In order to attain the condition that material 1 be in compression at the interface, we must have

\[ \sigma_1 = E_1 \left( \varepsilon_1^{\text{tot}} - \alpha_1 \Delta \theta \right) \leq 0 \Rightarrow \varepsilon_1^{\text{tot}} \leq \alpha_1 \Delta \theta \Rightarrow \frac{(\alpha_2 - \alpha_1) \Delta \theta}{E_1 A_1 K} \leq 0, \]  

which arises from using Eq. (11). If \( \Delta \theta \leq 0 \), then

\[ \frac{(\alpha_2 - \alpha_1) \Delta \theta}{E_1 A_1 K} \geq 0. \]  

Thus, for a compressive state to be induced, we must have \( \alpha_2 \geq \alpha_1 \). Clearly, this is intuitive, since if the substrate contracts more it will compress (pull inwards) the outer layers. Next, we consider the more complex case which includes curvature changes and hence bending stresses.

### 3. Combined thermal, extensional and bending deformation

Referring to Fig. 3, there are three contributions to the strain. The strain, due to the extensional load in material 1, is

\[ \varepsilon_1^E = \frac{F_1}{E_1 A_1} \]  

and in material 2

\[ \varepsilon_2^E = \frac{F_2}{E_2 A_2}. \]  

The strain, due to bending (curvature changes), in material 1 is

\[ \varepsilon_1^b = -\frac{M_1 y_1}{E_1 I_1} \]  

and in material 2 is

\[ \varepsilon_2^b = -\frac{M_2 y_2}{E_2 J_2}. \]  

The thermal contraction contribution for material 1 is

\[ \varepsilon_1^h = \alpha_1 \Delta \theta \]  

and for material 2 is

\[ \varepsilon_2^h = \alpha_2 \Delta \theta. \]

For this simple model, at the perfectly bonded interface, one equates the total normal strain at the interface to yield

\[ \varepsilon_2^{\text{tot}} = -\left( \frac{\alpha_2 - \alpha_1) \Delta \theta}{E_2 A_2 K} \right) + \varepsilon_2^h. \]  

In the present case, where there is no bending, the layer has a uniform stress with the one-dimensional strength-of-materials solution.
\[ \epsilon_1^i + \epsilon_1^b + \epsilon_1^o = \epsilon_2^i + \epsilon_2^b + \epsilon_2^o. \]  
(21)

which gives, at \( y_1 = y_1^{int}\) and \( y_2 = y_2^{int}\),

\[ \frac{F_1}{E_1A_1} - \frac{M_1y_1^{int}}{E_1I_1} + \alpha_1 \Delta \theta = \frac{F_2}{E_2A_2} - \frac{M_2y_2^{int}}{E_2I_2} + \alpha_2 \Delta \theta. \]  
(22)

Also, the curvatures of the strips must be equal

\[ \frac{1}{R_1} = \frac{M_1}{E_1I_1} = \frac{1}{R_2} = \frac{M_2}{E_2I_2} \Rightarrow M_1 = M_2 \frac{E_1I_1}{E_2I_2}. \]  
(23)

At a cross-section, statics dictates that the sum of the moments is zero (here, with summation about the interface)

\[ \sum M = M_1 + M_2 - F_1d_1^{int} + F_2d_2^{int} = 0, \]  
(24)

where \( d_1^{int} = |y_1^{int}| \) is the distance from the neutral axis of material 1 to the interface, and \( d_2^{int} = |y_2^{int}| \) is the distance from the neutral axis of material 2 to the interface. We must also have that the sum of the forces is zero

\[ \sum F = F_1 + F_2 = 0. \]  
(25)

Solving Eqs. (22)-(25) simultaneously for the 4 unknowns, \( F_1, F_2, M_1, \) and \( M_2 \) yields:

- For the force in material 1:
  \[ F_1 = \frac{(x_2 - x_1) \Delta \theta}{\Psi \Lambda}. \]  
(26)

- For the force in material 2:
  \[ F_2 = -\frac{(x_2 - x_1) \Delta \theta}{\Psi \Lambda}. \]  
(27)

- For the moment in material 1:
  \[ M_1 = \frac{(x_2 - x_1) \Delta \theta \ E_1I_1}{\Psi}. \]  
(28)

- For the moment in material 2:
  \[ M_2 = \frac{(x_2 - x_1) \Delta \theta}{\Psi}, \]  
(29)

where

\[ \Lambda = \frac{d_1^{int} + d_2^{int}}{1 + \frac{E_1I_1}{E_2I_2}} \]  
(30)

and

\[ \Psi = \frac{1}{\Lambda E_1A_1} + \frac{d_1^{int}}{E_1I_1} + \frac{1}{\Lambda E_2A_2} + \frac{d_2^{int}}{E_2I_2}. \]  
(31)

The total strain anywhere in material 1 is

\[ \epsilon_1^{tot} = \left( \frac{(x_2 - x_1)}{E_1A_1 \Psi \Lambda} - \frac{(x_2 - x_1) \ y_1}{E_2I_2} + \alpha_1 \right) \Delta \theta \text{ def } \gamma_1 \Delta \theta. \]  
(32)

and the total strain anywhere in material 2 is

\[ \epsilon_2^{tot} = -\left( \frac{(x_2 - x_1)}{E_2A_2 \Psi \Lambda} + \frac{(x_2 - x_1) \ y_2}{E_2I_2} + \alpha_2 \right) \Delta \theta \text{ def } \gamma_2 \Delta \theta. \]  
(33)

To meet the criteria that material 1 be in compression at the interface, we must have

\[ \sigma_1 = E_1(\epsilon_1^{tot} - \alpha_1 \Delta \theta) \leq 0 \Rightarrow \epsilon_1^{tot} \leq \alpha_1 \Delta \theta \Rightarrow \gamma_1 \Delta \theta \leq \alpha_1 \Delta \theta. \]  
(34)

If \( \Delta \theta \leq 0 \), then

\[ \gamma_1 \Delta \theta \leq \gamma_1 \alpha_1 \Delta \theta \Rightarrow \gamma_1 \geq \alpha_1, \]  
(35)

which collapses, in the case of \( x_2 \geq x_1 \), to \( y_1^{int} = -d_1^{int} \)
\[
\frac{\Delta E_1 A_1 y_{int}^{\text{int}}}{E_1 I_2} \leq 1
\]

and in the case of \( \alpha_2 \leq \alpha_1 \), to

\[
\frac{\Delta E_1 A_1 y_{int}^{\text{int}}}{E_1 I_2} \geq 1.
\]  

4. Special cases and inverse problems

If we consider rectangular cross-sectional areas of the form \( A_1 = b_1 h_1 \) and \( A_2 = b_2 h_2 \), then \( I_1 = \frac{1}{12} b_1 h_1^3 \), \( I_2 = \frac{1}{12} b_2 h_2^3 \), \( y_{int}^1 = -d_{int}^1 = -\frac{h_1}{2} \) and \( y_{int}^2 = d_{int}^2 = \frac{h_2}{2} \), and substituting into Eq. (35), we have, in the case of \( \alpha_2 \leq \alpha_1 \)

\[
3 \left( 1 + \frac{h_2}{h_1} \right) \left( 1 + \frac{E_2}{E_1} \right) \left( \frac{b_2}{b_1} \right) \leq 1.
\]  

and in the case of \( \alpha_2 \leq \alpha_1 \)

\[
3 \left( 1 + \frac{h_2}{h_1} \right) \left( 1 + \frac{E_2}{E_1} \right) \left( \frac{b_2}{b_1} \right) \geq 1.
\]  

Thus, the material designer has a total eight free parameters for system with rectangular cross-sections, namely, four material parameters \( (\alpha_1, \alpha_2, E_1, E_2) \) and four geometrical parameters \( (h_1, h_2, b_1, b_2) \). In cases where it is unlikely that one can find a set of parameters that satisfy case where Eq. (38) or (39) are valid, one consider attempt to minimize the amount of violation by considering the following cost function (regardless of whether \( \alpha_1 > \alpha_2 \), or vice versa) that represents attaining a zero stress state at the interface for material 1

\[
\Pi(E_1, E_2, h_1, h_2, b_1, b_2) \overset{\text{def}}{=} \left( \frac{3 \left( 1 + \frac{h_2}{h_1} \right) \left( 1 + \frac{E_2}{E_1} \right) \left( \frac{b_2}{b_1} \right)}{2} - 1 \right)^2.
\]

A plot of \( \Pi \) is shown in Fig. 4. This allows one to determine appropriate combinations of the height of the deposition, thickness of the substrate and the thermo-elastic combinations of the substrate for a (neutral) zero-stress condition. Plots for parameter combinations which deliver compressive stress states can also be generated by adjusting the constant “1” on the righthand side of Eq. (38) or (39) with another appropriate constant.

Fig. 4. Left: the full nondimensional parameter space for the values of \( \Pi \). Right: orthogonal slices through parameter space. Clearly, blue regions are advantageous and red regions are not. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Remark 1. One could apply Newton’s method, by forming the Hessian and Gradient, and set up the following system of equations

\[
\begin{bmatrix}
\frac{\partial^2 E}{\partial x_1^2} & \frac{\partial^2 E}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 E}{\partial x_1 \partial x_n} \\
\frac{\partial^2 E}{\partial x_2 \partial x_1} & \frac{\partial^2 E}{\partial x_2^2} & \cdots & \frac{\partial^2 E}{\partial x_2 \partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2 E}{\partial x_n \partial x_1} & \frac{\partial^2 E}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 E}{\partial x_n^2}
\end{bmatrix}
\begin{bmatrix}
\Delta x_1 \\
\Delta x_2 \\
\vdots \\
\Delta x_n
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial E}{\partial x_1} \\
\frac{\partial E}{\partial x_2} \\
\vdots \\
\frac{\partial E}{\partial x_n}
\end{bmatrix}
\]

(41)

However, because the system may not necessarily have unique minima (II is nonconvex), the Hessian may not be positive definite, and thus nonconvex optimization techniques, may need to be used.

Remark 2. In the simplest case, where \( h_1 = h_2 \) and \( b_1 = b_2 \), in the case of \( x_2 \ll x_1 \), the criteria in Eq. (38) collapses to

\[ E_2 \leq 5E_1 \]

(42)

and in the case of \( x_2 \gg x_1 \), the criteria in Eq. (38) collapses to

\[ E_2 \geq 5E_1 \]

(43)

Specifically, the substrate must be at least five times stiffer than the print-line to retain a compressive state.

5. Summary

The simple analysis presented provides some general guidance on the material combinations that are the best choice to attain a compressive residual stress state. On a two dimensional surface, the criteria (if it even exists) is unclear, and is under investigation by the author. However, we note that a preliminary finite element analysis indicates that the nonconvex shapes of deposited films on substrates will be impossible to keep in compression throughout the curing film structure, due to the highly nonuniform states of stress. Finally, following up on the introductory comments on the type of technology that drives this analysis, we remark that, unfortunately, inkjet and electrohydrodynamic printing do not allow precise control over the structure of the printed lines. This often results in lines with scalloped edges or non-uniform widths, and offer only limited control over the height of the printed features (Ahmad, Rasekh, & Edirisinghe, 2010; Huang et al., 2003; Samarasinghe, Pastoriza-Santos, Edirisinghe, Reece, & Liz-Marzan, 2006; Sirringhaus et al., 2000). Recently, nanoimprint lithography has been proposed as a means of decreasing the feature size of patterned nanoparticles while allowing more precise control over the structure of the printed lines (Ko et al., 2007, 2008; Park et al., 2008). In this fabrication method, the nanoparticle inks are patterned by pressing with an elastomer mold and the particles are dried into their final configuration. While the resolution of nanoimprint lithography is an improvement over inkjet printing, there exists a residual layer on the substrate that must be etched away after patterning. Control over the height of features can be corrupted by capillary action between the mold and the drying ink, in particular along the length of longer features. Thus, as a possible alternative to nanoimprint lithography, nanoparticle self-assembly methods, based on capillary filling of photore sist templates have been proposed (Demko, Cheng, & Pisano, 2010), and appear to be promising. It is important to note that many of the processes involve evaporative cooling, and require a simultaneous heat and mass transfer analysis. A complete model and numerical simulation of this process is currently underway by the author.

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