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Technical note

Electromagnetic control of charged particulate spray systems—Models for planning the spray-gun operations

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HIGHLIGHTS

- Derived criteria to control charged particulate sprays using electromagnetic fields.
- Such criteria can be used in physically based computer simulations for such sprays.
- Coupled multi-physics aspects of such modifications are presented.
- These aspects are critical for identifying core issues to guide future research.

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ABSTRACT

Charged particulate spray systems are common in many industrial and manufacturing processes. Using externally applied electromagnetic fields – the dynamics of these particulate sprays can be altered to achieve improved functionality and access to spray-sites that are hard to reach. With such an alteration the spray-particulate dynamics can become non-intuitive – thereby motivating a physically based modeling strategy to plan the spray-gun operations and translate this into the actual spray deposition on the target surface. In this paper we use the dynamics of charged particles to construct a set of simple geometric arguments for the identification of the mapping between the spray-gun trajectory (on its plane of traversal) and the spray-deposit location (on the plane of the target-surface). The parametric dependence of the mapping on spray-gun operation parameters (comprising nozzle velocity and trajectory) and external magnetic fields (comprising field strength and the region of applied field) is discussed. The role of such arguments in constructing appropriate computer simulation frameworks is then illustrated through an example of a discrete element simulation. Sensitivity to process parameters like particulate size and spray-gas velocity are also characterized for a given applied field.

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1. Introduction

In this work we focus on industrial and manufacturing processes that involve a stream of loosely flowing charged particulates. These include a variety of processes like electrostatic spraying (see for example [1]), electrostatic powder coating processes (see for example [2,3]), and techniques like material removal by electrostatic scrubbing (see for example [4]). Directed streams of charge particulates are also finding application in combustion processes and biological drug delivery. As the particulates (or even charged droplets in many applications) travel from their release point, their trajectory can be further modified using external electromagnetic fields of appropriate strength. This provides a potentially useful and novel way of altering these spray-processes to achieve improved functionality (for example increased uniformity in spray deposit owing to repulsive forces between like-charged particulates). It can also be used to direct particulate sprays towards a target surface that would otherwise be difficult to access directly. While the trajectory planning of a robot arm for spray-guns is a widely researched topic (for an overview of which, see [5,6], and the work on shape deposition by [7], and [8])—the use of external electromagnetic fields would require these planning strategies to be augmented appropriately. With the recent advances in available computational resources and efficient algorithms, physically based modeling and simulation provide a viable alternative to perform parametric investigations for modifying particulate spray processes using electromagnetic fields, and provide valuable insights into their engineering and development, thereby allowing for an advancement of the state-of-art in guided charged particulate sprays. In this context, we focus here on developing simple geometric criteria to map the spray-gun trajectory (on its plane of traversal), and the spray-deposit location—and

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linking such criteria to the computer aided engineering of such processes. In precise terms, the contribution of this work is to investigate the following aspects:

- the derivation of simple design criteria to guide the particles using appropriate spray-gun parameters and electromagnetic fields.
- the utility of such criteria in augmenting computer simulations of charged particulate sprays to aid process engineering and analysis.

To this end, we present a brief discussion on the dynamics of charged particles in electromagnetic fields and present some simple geometric arguments for the mapping in Sections 2 and 3. These models are implemented into a discrete element based simulation of charged sprays in Section 4 with a discussion of the coupled multi-physics aspects of such process modifications so as to identify core issues to guide future research. The focus here is on physical models and their use in computer simulation tools, and not large-scale simulations cross-validated with experimentation. This work is part of an ongoing effort to develop a general-purpose computer simulation framework using discrete element methods for charged particulate flows.

2. Modeling the particle motion

The motion of a charged particle in an electromagnetic field is governed by the classical Lorentz Force, which is given in terms of the strengths of the electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$ respectively) as:

$$\mathbf{F}_{\text{lorentz}} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right). \tag{1}$$

It is evident that if the velocities and electric fields are decomposed in directions parallel and perpendicular to the magnetic field (e.g. $\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$ etc.), the effective motion equations for a single particle can be decomposed as (see also [9] for detailed solutions):

$$m\frac{d\mathbf{v}_{\parallel}}{dt} = q\mathbf{E}_{\parallel}, \qquad m\frac{d\mathbf{v}_{\perp}}{dt} = q\left(\mathbf{E}_{\perp} + \mathbf{v}_{\perp} \times \mathbf{B}\right) \tag{2}$$

which is a consequence of the Lorentz force having the $\mathbf{v}\times\mathbf{B}$ term. If we consider a Cartesian coordinate system (with $\mathbf{v} = v_x\mathbf{x} + v_z\mathbf{z}$, and $\mathbf{B} = B\mathbf{y}$) and assume furthermore that $\mathbf{E} = \mathbf{0}$, the motion equations can then be written as:

$$m\frac{dv_x}{dt} = -qBv_z, \qquad m\frac{dv_z}{dt} = qBv_x. \tag{3}$$

With a given initial velocity $\mathbf{v}(0) = v_x(0)\mathbf{x} + v_z(0)\mathbf{z}$, the above can be multiplied by v_x and v_z respectively to get a form of conservation of mechanical energy as follows:

$$mv_x\frac{dv_x}{dt} + mv_z\frac{dv_z}{dt} = 0 \Rightarrow \frac{d}{dt}\frac{m}{2}\left(v_x^2 + v_z^2\right) = 0. \tag{4}$$

Furthermore, taking second derivatives of particle velocity equations in Eq. (3) we get:

$$\frac{d^2v_x}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_x, \qquad \frac{d^2v_z}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_z. \tag{5}$$

The solution to this system of ordinary differential equations with given initial velocity $\mathbf{v}(0)$ and energy conservation as in Eq. (4) can be given as follows:

$$v_x(t) = |\mathbf{v}(0)|\sin\left(\omega_c t + \phi_0\right) \tag{6}$$

$$v_z(t) = |\mathbf{v}(0)|\cos\left(\omega_c t + \phi_0\right) \tag{7}$$

with $\omega_c = qB/m$ and $\tan\phi_0 = v_x(0)/v_z(0)$. The frequency $\omega_c/2\pi$ is what is referred to as the cyclotron frequency. Integrating further with respect to time, the coordinates of the particle can be obtained as follows:

$$x(t) = \frac{-|\mathbf{v}(0)|}{\omega_c}\cos\left(\omega_c t + \phi_0\right) + X_0 \tag{8}$$

$$z(t) = \frac{|\mathbf{v}(0)|}{\omega_c}\sin\left(\omega_c t + \phi_0\right) + Z_0 \tag{9}$$

with X_0 and Z_0 being defined as follows:

$$X_0 = x(0) + \frac{|\mathbf{v}(0)|}{\omega_c}\cos\phi_0, \qquad Z_0 = z(0) - \frac{|\mathbf{v}(0)|}{\omega_c}\sin\phi_0. \tag{10}$$

The particle trajectory can be given as

$$(x - X_0)^2 + (z - Z_0)^2 = \left(\frac{|\mathbf{v}(0)|}{\omega_c}\right)^2 \tag{11}$$

which is a circle with its center at (X_0, Z_0), and radius of $R_c = \frac{|\mathbf{v}(0)|}{\omega_c} = \frac{m|\mathbf{v}(0)|}{qB}$. The radius R_c is what is commonly referred to as the Larmor radius. Furthermore, the existence of a non-zero electric field induces additional components of motion—a cycloidal translation along the axis of the circle due to $\mathbf{E}_{\perp}$, and pure translation due to $\mathbf{E}_{\parallel}$. There is also additional motion in form of an induced drift from the circular trajectory due to additional (non-electromagnetic) forces acting simultaneously. For the scope of this work, these additional components of the motion are not discussed here.

3. Models for spray trajectory planning

The discussion presented in Section 2, can now be applied to derive geometric criteria for bending the trajectory of a stream of charged particulates. To develop this idea, refer to Fig. 1—wherein a charged particle is bent by an angle α by allowing it to pass through a region of applied magnetic field $B\mathbf{y}$ over a region of width d (depicted in gray). Inside the region of applied magnetic field—the dynamics of this particle can now be exactly represented by the Eqs. (6)–(9), and (11). Using simple geometric arguments for the arc of this circular path (as presented in Fig. 1), it can also be shown that the angle subtended by the circular arc of the trajectory at the center is also α. Since the original and the final trajectories are tangential to the circle, the angle can be represented as:

$$\sin\alpha = \frac{d}{R_c} = \frac{dqB}{m|\mathbf{v}(0)|}. \tag{12}$$

In order for a spray based deposition, the spray is ideally kept normal to the target surface (see for example [10], and [6] for discussions on spray incidence). Therefore, in order to bend the spray jet to be able to hit the target surface normally—the angle α should match that of the normal of the target surface with the horizontal (note that it is assumed that the distance of an edge or line of the part geometry from the spray-gun plane origin is known):

$$\sin\alpha = -\hat{\mathbf{N}}_{\text{target}} \cdot \mathbf{x} = dqB/m|\mathbf{v}(0)|. \tag{13}$$

This provides a geometric design criterion, that relates the spray-velocity and the applied magnetic field parameters to the target surface orientation—if the latter is known, the appropriate spray-conditions can be decided. Furthermore, if the mapping between the target surface geometry and the spray particulate release plane is characterized using a mapping function $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X})$ as represented schematically in Fig. 2, then the normal vector of a segment of the target surface can be mapped correspondingly using the classical Nanson's formula as:

$$\hat{\mathbf{N}}_x dA_x = J[\mathbf{F}]^{-T}\hat{\mathbf{N}}_X dA_X \tag{14}$$

where dA_x and dA_X are the magnitudes of area of the discretized mesh elements in the two coordinate systems as in Fig. 2, and the second order tensor $[\mathbf{F}] = \partial\mathbf{x}/\partial\mathbf{X}$ is the deformation gradient for the mapping, and $J = \det[\mathbf{F}]$.

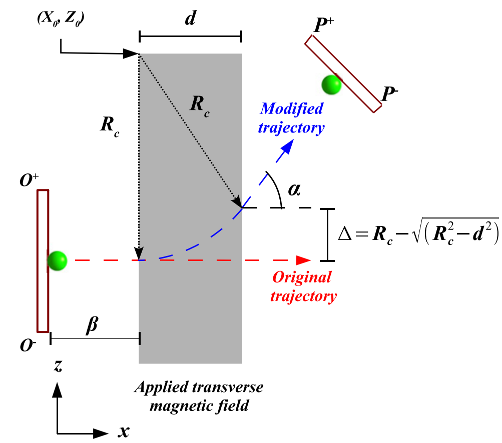

Fig. 1. Schematic demonstrating the deflection of a single charged particle upon traversing a region of transverse electromagnetic field.

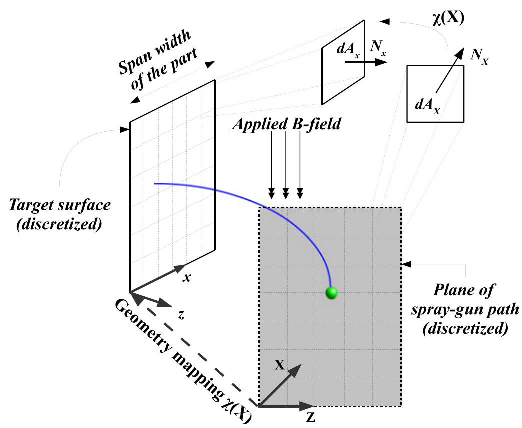

Fig. 2. Schematic representation of the geometric mapping between target surface geometry and the plane of traversal of spray gun. The mapping of surface normals for a single representative element is highlighted.

For planning the spray-gun operations it is also of interest to know the extent of the spray-gun motion in its plane of traversal such that the deviated particulate trajectory can still impact the target surface. For this, a simple geometric argument on projecting the extents of a horizontal slice of the target-surface (i.e. a section at a certain y-coordinate location for our discussion), is considered. Referring to Fig. 1, a ray emanating from point P^+ of a section $\overline{P^+P^-}$ of the target surface, can be represented as:

$$z - z^+ = \left(x - x^+\right) \tan\alpha$$

where, x^+, z^+ are the coordinates of the planar segment $\overline{P^+P^-}$. The intersection of this ray with the plane at $x = \beta + d$ is given now by:

$$x = \beta + d, \qquad z = z^+ + \tan\alpha \left(\beta + d - x^+\right).$$

This point can now be mapped back onto the original spray-gun plane by considering the deflection Δ (refer Fig. 1) in the trajectories due to the applied magnetic field. This gives us (on the plane of the gun, $x = 0$):

$$z_0^{\pm} = z^{\pm} + \tan\alpha \left(\beta + d - x^{\pm}\right) + R_c - \sqrt{R_c^2 - d^2}. \tag{15}$$

This provides another geometric design criterion for the boundaries within which the spray gun is required to traverse ($\overline{O^+O^-}$), for a spray being deflected by an applied electromagnetic field towards a segment $\overline{P^+P^-}$ of the target surface.

The expressions derived here, can now be used in conjunction with finite element mesh-based representation of target surfaces

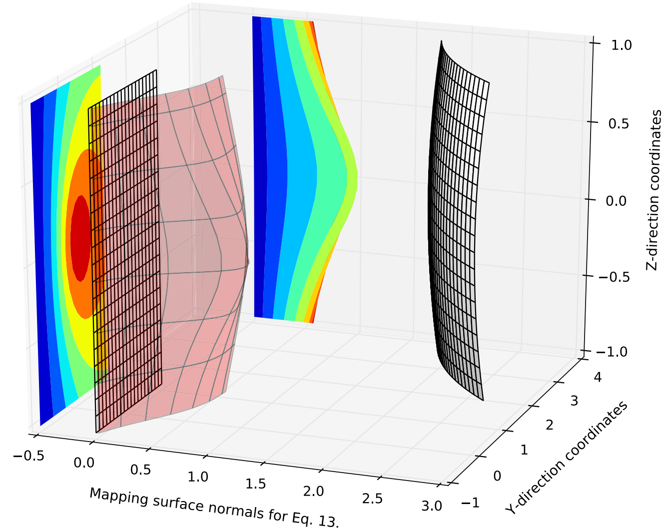

Fig. 3. An illustration of using target surface geometries in conjunction with Eq. (13). The spray-gun plane and target-surface mesh (shown as black and white wireframes) are overlaid with the variation of $\sin(\alpha)$, along with projected contour representations of the same (contour levels going from 0 to 1).

to set appropriate parameters (i.e. gun exit velocity, applied magnetic field, etc.) in order to modify a particulate spray for an application. The coordinates and the surface normals obtained from the meshed geometry will be inputs for Eqs. (13)–(15). While a detailed implementation of a spray simulation using a mesh-discretized geometry is beyond the scope of this work, a simple example demonstrating the variation of $\sin(\alpha)$ for a paraboloid target surface geometry is presented in Fig. 3. We remark furthermore that the angle α actually can be varied across different sections of the target surface—to simulate the effect of a variable pitch angle or a camber on the part to be sprayed (something common in a lot of industrial turbo machinery). The expressions derived here also provide models for constructing numerical spray-process simulation frameworks for electromagnetic control—a more detailed discussion and an illustration of which is the focus of the next section.

4. Simulation

The problem of deviating a charged particulate spray is now analyzed using a discrete element simulation framework – using the design criteria derived in Eqs. (13) and (15). Discrete element simulations in general comprise the numerical solution of the motion equations of discrete computational elements – usually spherical in geometry. For a broad overview on such methods in general see [11]. Particularly for applications in flowing particulate systems see [12,13], and for more specific review of such techniques for charged particulate systems, see [14].

To construct the simulations, an ensemble of spray particulates are generated by using the random sequential addition of hard-spheres (as presented in [15]) in a cuboidal region of space (see Fig. 4 for a representation of the computation domain). Such algorithms are suitable for rapid generation of ensembles for moderate packing densities. We consider the motion of these particles through an applied transverse magnetic field as depicted in Figs. 1 and 2. It is reasonable to assume that all particles are charged alike—and therefore, there are repulsion forces acting between particles that reduce chances of collisions. Furthermore, it is reasonable to assume that particle response times to any applied fluid (spray-gas) velocities will be low (since in reality response time $\propto d_p^2$ and spray applications have small particle sizes (d_p)). Therefore, placing the transverse field such that β corresponds to a region very close to the core of the spray-gun jet, it can be assumed

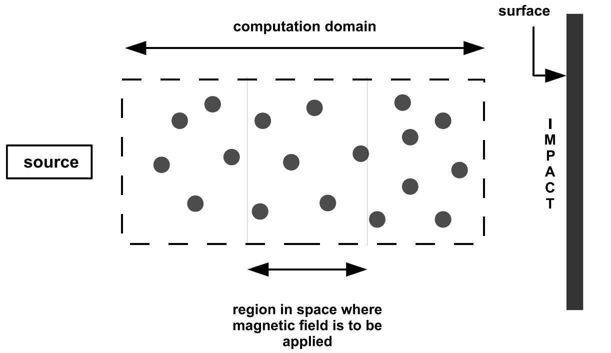

Fig. 4. A schematic of the typical computational domain for spray simulations. The region of applied fields is marked—however, the deflected trajectories are not shown.
Source: The spray-gun nozzle.

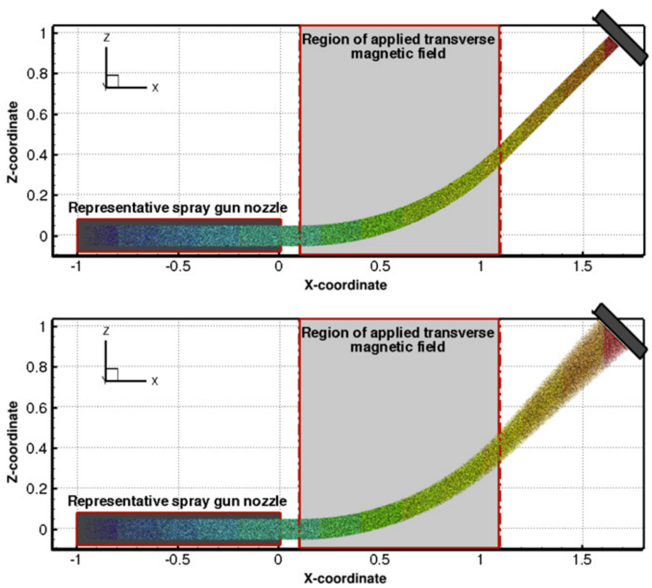

Fig. 5. Simulation results for the complete particle trajectories of a charged particulate spray—without any variability in diameter and exit velocity (*top*), and with a coefficient of variation of 0.1 in exit velocity and diameter (*bottom*). The particles are marked with a pseudocolor based on their location to make the spray trajectories visible.

that the particles enter the magnetic region with a velocity almost equal to the nozzle gas velocity. The motion of the jet depends dominantly now on the Lorentz forces, up until the impact with the target surface. This particular argument of Lorentz forces dominating and effects of Coulombic interactions being less significant will be revisited at a later section.

The target surface impact is resolved using a geometric overlap check between a spherical particle and a plane, as follows:

$$\frac{|ax_p + by_p + cz_p + d_0|}{\sqrt{a^2+b^2+c^2}} < \frac{d_p}{2} \tag{16}$$

with a, b, c, d_0 being the coefficients of the equation of the target surface, and x_p, y_p, z_p is the coordinate of the center of a particle at a certain instant of time. Upon impact we assume that all impacting particles stick to the surface—since the development of an appropriate criteria for deposition would be beyond the scope of this paper.

The resultant trajectories of a sample simulation process with 50 µm radius Zirconia powder are presented in Fig. 5 (*top*). The jet velocity and the field strengths are correlated using Eq. (12), and a very small cross-span velocity (x-component) is applied to move the gun across the spray-plane. It was observed from the simulations that this cross-span velocity is for all practical purposes very small compared to the jet exit velocity—and hence play a minimal role in modifying the particle trajectories as they enter the magnetic field.

It is noted that the powder particle sizes and the particle exit velocities from the gun may have variabilities in them, the effect of which needs to be also investigated for such a modification to the spray processes. For visualizing this—a sample simulation for the same Zirconia powder with a coefficient of variation of 0.1 in both particle size and exit velocity has been presented in Fig. 5 (*bottom*). It is evident that while for the homogeneous particle size and velocities the spray jet gets concentrated prior to impact with target surface—the added variabilities cause the jet to spread out, thereby causing a potential reduction in the deposit pattern efficiency. Therefore, it is worthwhile, from a process engineering perspective to seek a simple analysis of the effect of the variabilities. In order to further quantify this effect we rewrite the explicit dependence on the particle diameter in Eq. (12):

$$\alpha = \sin^{-1}\left[\frac{6dB}{\pi\rho_p}\frac{q}{d_p^3 V}\right]. \tag{17}$$

From this the following simple estimate for the variability of the deviation angle (characterized by a variance σ_α) can be formulated:

$$\sigma_\alpha = \left(\frac{\partial\alpha}{\partial d_p}\right)^2_{\mu_p,\mu_v}\sigma_p + \left(\frac{\partial\alpha}{\partial v}\right)^2_{\mu_p,\mu_v}\sigma_v + \left(\frac{\partial\alpha}{\partial d_p}\frac{\partial\alpha}{\partial v}\right)_{\mu_p,\mu_v}\sigma_{p,v} \tag{18}$$

$$= \frac{1}{1-\left(\sin^2\alpha\right)_\mu}\left[\frac{6dBq}{\pi\rho_p}\right]^2\left(\frac{1}{\mu_v^2\mu_p^6}\right) \times \left(\frac{9\sigma_p}{\mu_p^2} + \frac{\sigma_v}{\mu_v^2} + \frac{3\sigma_{p,v}}{\mu_v\mu_p}\right) \tag{19}$$

where the variability in the feed powder size is characterized by a variance σ_p and that in the exit velocities is characterized by a variance σ_V. Their joint variability is characterized by the covariance $\sigma_{p,v}$. The corresponding average particulate size and exit velocities are given by μ_p, μ_v respectively, and $\left(\sin^2\alpha\right)_\mu$ is the value obtained by plugging in the average values μ_p, μ_v in Eq. (12). For most practical applications $\mu_p \approx \mathcal{O}$ (µm), and $\mu_v \approx \mathcal{O}$ (m/s). For similar levels of variability in the particle size and exit velocity—from Eq. (19) it follows that the deflected beam will tend to get more sensitive to changes in the particle size, hence getting more diffuse for a more heterogeneously sized particulate ensemble. We remark that the case of $\sigma_{p,v} \approx 0$ corresponds to uncorrelated powder size and exit velocities—which corresponds to applications where they originate from different sources and the powder is thereafter released into the gas stream. It also corresponds to applications where variability in gas velocities is due to the flow structure, and is independent of the particles' dynamics inside the region of applied external field. Furthermore, it must be noted that in order to get the exact extent to which the jet gets concentrated or diffused just before impact, a detailed resolution of all forces should be taken into account, since very close to the surface, with more closely packed particulates, the Coulomb interactions may cause significant contributions to the shape of the spray.

5. Conclusions and future work

The computer aided analysis of a charged spray process has been discussed with the objective of using electromagnetic fields to modify the spray trajectory. It was shown that the spray parameters and the target surface geometry are related to the magnetic field parameters through Eq. (13), and the corresponding spray-gun path coordinates are then linked using the criteria in Eq. (15). The properties of the spray-gun were lumped into the exit velocity from the nozzle ($\mathbf{V}(0)$), and the trajectory of the gun in its plane of traversal ((x, z)). The properties of the applied field were lumped into the field strength and geometry (B, d respectively). The utility of such criteria in augmenting computer simulation frameworks for process analysis was demonstrated using a discrete element simulation framework, along with a quantification of the uncertainties associated with the particulate size and velocities. These considerations together provide a physically based model for computer aided engineering of an electromagnetically modified charged particulate spray. The optimization of the process parameters has not been addressed—but the discussions presented can be readily extended to construct optimal parameters and applied fields.

Physically, there are two relevant aspects of the modeling that need to be touched upon in conclusion to motivate outlining the future research efforts in terms of simulation and experimental validation. The first is the role of an external non-electromagnetic force that leads to a drift motion of the particulates. This force can be, for example, due to gravity or due to fluid drag from the surrounding media. A complete resolution of these forces lead to a coupled system of ordinary differential equations that need to be numerically solved for each particle in the ensemble—which is more amenable to a large-scale process simulation than a derivation of a basic process design criteria for planning the spray-gun parameters.

The second aspect is that of the dominance of the Lorentz force over Coulombic interactions. Mathematically speaking, this entails the observation of a scaling argument between the relative magnitudes of these two forces:

$$q|V|B \gg \frac{q^2}{4\pi\epsilon r^2} \tag{20}$$

where r is a characteristic inter-particle separation for the ensemble of particulates. It can be assumed that $r = \gamma d_p$, where γ is an appropriate scaling measure of the separation between particles in terms of particle sizes. For most practical applications $d_p \approx \mathcal{O}$ (20–80 μm), and $q \approx \mathcal{O}$ (10–100 μC). Plugging these orders of magnitude in, it can be seen that the argument on the relative magnitudes will hold only for very small γ, or very high $|V|$. The factor γ correlates inversely to the volume-fraction of the particulate jet—which means that for a low volume-fraction ensemble of particles, the average inter-particle separation is high. This indicates that the Coulomb interactions are weaker for low-volume fraction ensembles traveling at high-velocities—which is precisely the regime that many of the commercial spray applications fall under.

Therefore, while the design criteria for spray-gun operations presented in paper are appropriate for most spray based applications—detailed resolution of the spray behavior under the action of combined fluid, Coulomb, and Lorentz forces will be only possible to be dealt with using large-scale computer simulations, for which the design equations presented here would be used as inputs.

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