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Short Communication

On the reduction of heat generation in lubricants using microscale additives

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ABSTRACT

This work is concerned with the identification of microscale properties of additives for base lubricants in order to reduce heat generation. An application of specific interest is the thin film lubrication of bearings. In order to isolate the thermal effects in the fluid film, we assume that the bearing and housing are insulated. A relation for the temperature rise in the fluid film between the bearing and housing is developed as a function of the rotation speed, the viscosity of the base lubricant and properties of the additives, namely (1) their viscosities, (2) their mass density, (3) their heat capacity and (4) volume fraction, which are free design parameters. Nondimensionalization of the developed relations allows for the construction of a design parameter space which can identify desirable parameter combinations that deliver a target value of heat generation reduction and simultaneously deliver the appropriate overall viscosity of the modified lubricant mixture.

1. Simple fluid profiles

The "functionalization" or "tailoring" of solid and fluid materials by the addition of fine-scale material is a process that has a long history in engineering. The usual approach is to add particulates that possess a desired property to modify (enhance) a base (binder) material. There exist several methods to predict the resulting effective properties of materials with embedded particulates, dating back to well over a century to, for example, Maxwell (1867, 1873), Rayleigh (1892) and, specifically for effective viscosity characterizations, to Einstein (1906, 1911). For a thorough analysis of many of such methods, see Torquato (2001), Jikov, Kozlov, and Olenik (1994), Hashin (1983) and Nemat-Nasser and Hori (1999) for mechanics oriented treatments and (Ghosh, 2011a; Ghosh & Dimiduk, 2011b; Zohdi & Wriggers, 2008) for computational aspects. For a recent review of general effective viscosity models, see Abedian and Kachanov (2010) and Sevostianov and Kachanov (2012). We note that a wide range of additives are possible to modify lubricant properties; for example, see Wu, Tsui, and Liu (2007) for extensive experiments.

Our interest in this work is to identify material parameters for additives in order to modify lubricant properties with the goal of reducing heat generation in fluid films. An application of specific interest is the lubrication of bearings. Since the clearance between the bearing and housing are extremely small, the fluid flow profile is assumed to be spatially linearly-varying (radially, see Fig. 1, driven by the bearing, assuming concentric circular Couette flow)

$$v_x = \frac{R\omega}{\delta} y, \tag{1.1}$$

where δ is the film thickness and the other velocity components are zero, $v_y = v_z = 0$. We focus only on the viscous stresses, and ignore all other contributions, thus the stress tensor reduces to (assuming a Newtonian fluid)

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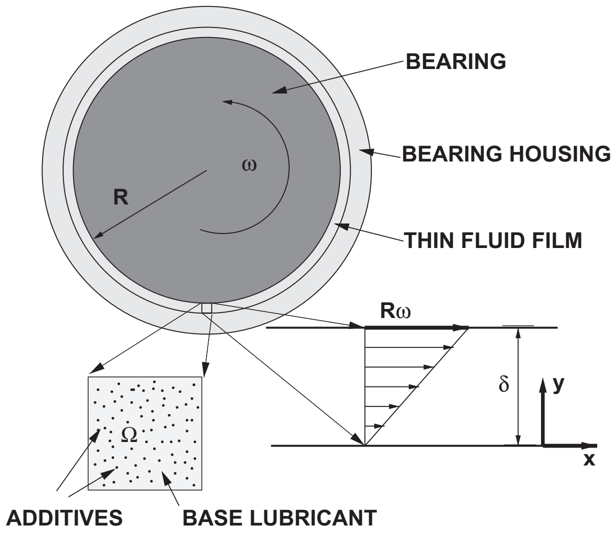

Fig. 1. Idealized fluid profile in a thin film.

$$\boldsymbol{\sigma} = 2\mu^*\boldsymbol{D} = \begin{bmatrix} 0 & \mu^*\frac{R\omega}{\delta} & 0 \\ \mu^*\frac{R\omega}{\delta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1.2}$$

where $\boldsymbol{D} = \frac{1}{2}(\nabla_{\boldsymbol{x}}\boldsymbol{v} + (\nabla_{\boldsymbol{x}}\boldsymbol{v})^T)$ and $\boldsymbol{\sigma}$ is Cauchy stress.

2. Heat generation

In order to focus on the properties of the lubricant, we assume that there is no heat transfer between the lubricant and the bearing and bearing housing. The key quantity of interest here is the amount of heat generated, calculated from first law of thermodynamics,

$$\rho^*\dot{w} - \boldsymbol{\sigma} : \nabla\boldsymbol{v} + \nabla\cdot\boldsymbol{q} = 0. \tag{2.1}$$

In Eq. (2.1), ρ^* is effective mass density, w is the stored energy per unit mass and $\boldsymbol{q}$ is heat flux, for example, due to conduction. If we assume that the temperature, T, is uniform in the film, throughout the thickness, with $w = C^*T$, where C^* is the effective heat capacity, volume averaging yields

$$\langle\rho C T\rangle_\Omega = \langle\rho C\rangle_\Omega\langle T\rangle_\Omega = \rho^* C^* T, \tag{2.2}$$

where

$$\langle(\cdot)\rangle_\Omega = \frac{1}{V_\Omega}\int_\Omega(\cdot)\,d\Omega, \tag{2.3}$$

where Ω is the domain over which the averaging takes place (Fig. 1), V_Ω is the corresponding volume and $\langle T\rangle_\Omega = T$, due to the uniformity of T. Furthermore, the heat capacity can be written as

$$\langle\rho C\rangle_\Omega = \rho^* C^* = v_1\rho_1 C_1 + v_2\rho_2 C_2, \tag{2.4}$$

where subscript 1 denotes the base lubricant (phase 1) and subscript 2 indicates the additives (phase 2). Also, due to the linear velocity profile, $\boldsymbol{D}$ is uniform, thus $\langle\boldsymbol{D}\rangle_\Omega = \boldsymbol{D}$ and

$$\langle 2\mu\boldsymbol{D}\rangle_\Omega = \langle 2\mu\rangle_\Omega\langle\boldsymbol{D}\rangle_\Omega = 2\mu^*\langle\boldsymbol{D}\rangle_\Omega = 2\mu^*\boldsymbol{D} = \langle\boldsymbol{\sigma}\rangle_\Omega, \tag{2.5}$$

where

$$\mu^* = v_1\mu_1 + v_2\mu_2, \tag{2.6}$$

where μ_1 and μ_2 are the dynamic viscosities of the two phases, and v_1 and v_2 are the corresponding volume fractions ($v_1 + v_2 = 1$). Since we have assumed uniformity of the temperature field, there is no angular dependence nor temperature variation through the film thickness, and if we further assume Fourier's law of conduction ($\boldsymbol{K}$ is the conductivity) holds, $\boldsymbol{q} = -\boldsymbol{K}\cdot\nabla_{\boldsymbol{x}}T$, then $\boldsymbol{q} = \boldsymbol{0}$, and we have

$$\rho^* C^* \frac{dT}{dt} = \langle \boldsymbol{\sigma} : \nabla \boldsymbol{v} \rangle_\Omega = \mu^* \frac{R^2 \omega^2}{\delta^2}. \tag{2.7}$$

The time derivative of the temperature can we decomposed into the stationary and convective contributions as follows

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \nabla_{\boldsymbol{x}} T \cdot \boldsymbol{v}, \tag{2.8}$$

thus, since $\nabla_{\boldsymbol{x}} T = \mathbf{0}$,

$$\rho^* C^* \frac{\partial T}{\partial t} = \mu^* \frac{R^2 \omega^2}{\delta^2}. \tag{2.9}$$

3. Inverse problem of lubricant design

Now we focus on identifying the parameters needed for a target rate of heating (H^{tar}), defined as

$$\frac{\partial T}{\partial t} = \frac{\mu^*}{\rho^* C^*} \frac{R^2 \omega^2}{\delta^2} \stackrel{\text{def}}{=} H^{tar}. \tag{3.1}$$

The heating rate generated by a pure phase 1 (unmodified "base" lubricant) is given by

$$\frac{\partial T}{\partial t} = \frac{\mu_1}{\rho_1 C_1} \frac{R^2 \omega^2}{\delta^2} \stackrel{\text{def}}{=} H^1. \tag{3.2}$$

Defining the target heat production in terms of the pure phase 1 (base lubricant) production yields

$$H^{tar} = \Phi H^1, \tag{3.3}$$

where $0 \leqslant \Phi \leqslant 1$. This yields

$$\frac{\mu^*}{\rho^* C^*} \frac{R^2 \omega^2}{\delta^2} \stackrel{\text{def}}{=} H^{tar} = \Phi H^1 = \Phi \frac{\mu_1}{\rho_1 C_1} \frac{R^2 \omega^2}{\delta^2}, \tag{3.4}$$

thus

$$\frac{\mu^*}{\rho^* C^*} = \Phi \frac{\mu_1}{\rho_1 C_1}. \tag{3.5}$$

Solving for the volume fraction, and writing all terms in nondimensional form, yields

$$v_2 = \frac{1 - \Phi}{\left(1 - \frac{\mu_2}{\mu_1}\right) - \Phi\left(1 - \frac{\rho_2 C_2}{\rho_1 C_1}\right)}, \tag{3.6}$$

which defines a surface for a given Φ in terms of nondimensional parameters, $v_2, \Lambda_\mu \stackrel{\text{def}}{=} \frac{\mu_2}{\mu_1}$ and $\Lambda_{\rho C} \stackrel{\text{def}}{=} \frac{\rho_2 C_2}{\rho_1 C_1}$. This surface gives design alternatives for the reduction of heat production, as shown in Fig. 2. The resulting effective viscosity is

$$\mu^* = \left(1 - \frac{1 - \Phi}{(1 - R_\mu) - \Phi(1 - R_\rho)}\right)\mu_1 + \left(\frac{1 - \Phi}{(1 - R_\mu) - \Phi(1 - R_\rho)}\right)\mu_2, \tag{3.7}$$

and is shown in Fig. 3.

Remarks: Because it is unlikely that one can find a set of parameters that satisfy the desired heat reduction and a desired effective viscosity simultaneously for all situations, one can formulate a cost function that balances the goals of heat rate reduction with a viscosity constraint, for example, given by

$$\Pi(v_2, \Lambda_\mu, \Lambda_{\rho C}) \stackrel{\text{def}}{=} w_1 \left\| \frac{\Phi - \Phi^{des}}{\Phi^{des}} \right\| + w_2 \left\| \frac{\mu^* - \mu^{*,des}}{\mu^{*,des}} \right\|. \tag{3.8}$$

An example of parameter space for $\Phi^{des} = 0.5$ and $\mu^{*,des} = 0.5\mu_1$, with weights $w_1 = w_2 = 1$, is shown in Fig. 4. One could apply Newton's method, by forming the Hessian and Gradient and solve for $i = 1,2,\ldots,N$,

$$\begin{bmatrix} \frac{\partial^2 \Pi}{\partial v_2^2} & \frac{\partial^2 \Pi}{\partial v_2 \partial \Lambda_\mu} & \frac{\partial^2 \Pi}{\partial v_2 \partial \Lambda_{\rho C}} \\ \frac{\partial^2 \Pi}{\partial \Lambda_\mu \partial v_2} & \frac{\partial^2 \Pi}{\partial \Lambda_\mu^2} & \frac{\partial^2 \Pi}{\partial \Lambda_\mu \partial \Lambda_{\rho C}} \\ \frac{\partial^2 \Pi}{\partial \Lambda_{\rho C} \partial v_2} & \frac{\partial^2 \Pi}{\partial \Lambda_{\rho C} \partial \Lambda_\mu} & \frac{\partial^2 \Pi}{\partial \Lambda_{\rho C}^2} \end{bmatrix} \begin{bmatrix} \left(v_2^{i+1} - v_2^i\right) \\ \left(\Lambda_\mu^{i+1} - \Lambda_\mu^i\right) \\ \left(\Lambda_{\rho C}^{i+1} - \Lambda_{\rho C}^i\right) \end{bmatrix} = - \begin{bmatrix} \frac{\partial \Pi}{\partial v_2} \\ \frac{\partial \Pi}{\partial \Lambda_\mu} \\ \frac{\partial \Pi}{\partial \Lambda_{\rho C}} \end{bmatrix}. \tag{3.9}$$

However, because the system may not necessarily have unique minima (Π is nonconvex), the Hessian may not be positive definite, and thus nonconvex optimization techniques, such Genetic Algorithms (Zohdi, 2003), may need to be used.

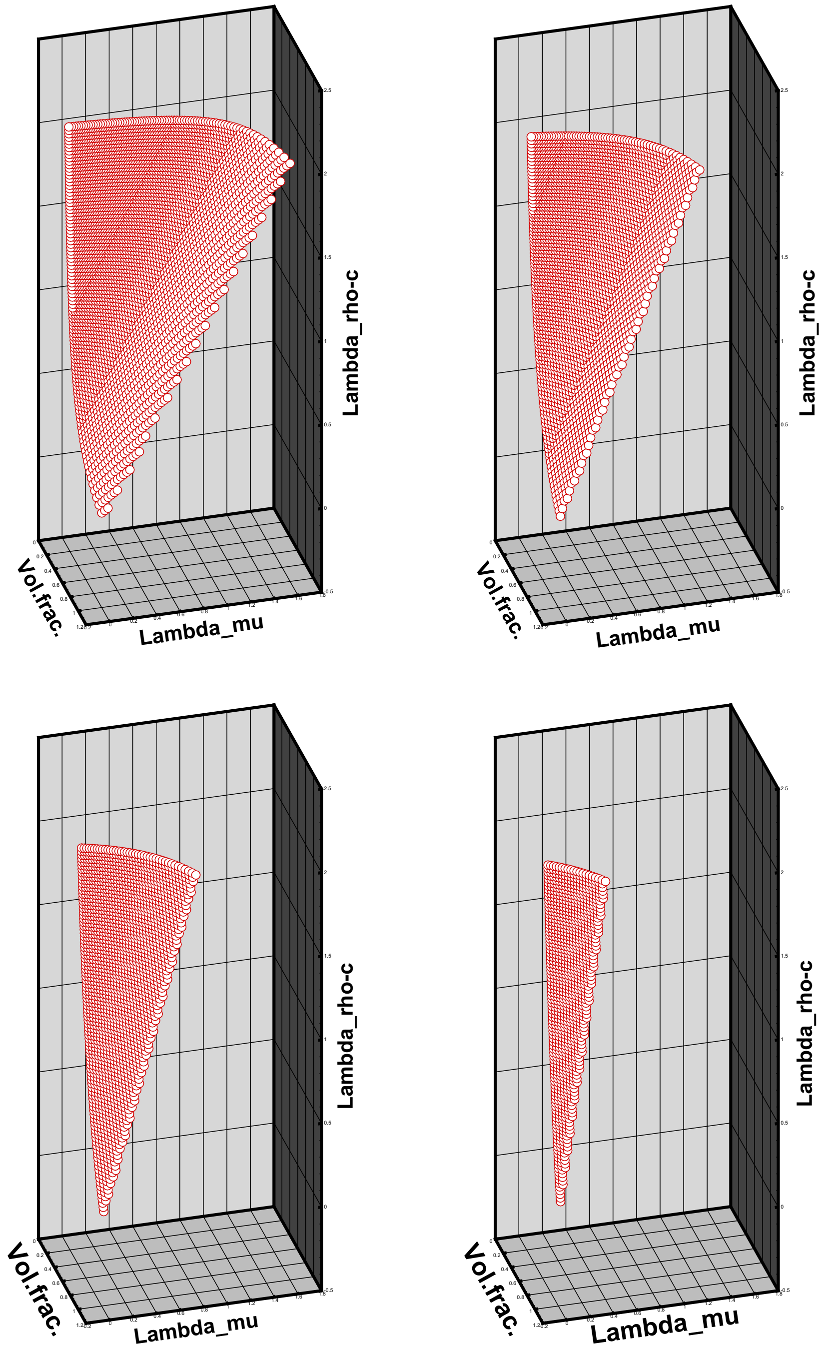

Fig. 2. Starting from left to right and top to bottom, the nondimensional surfaces for $\Phi = 0.8$, $\Phi = 0.6$, $\Phi = 0.4$ and $\Phi = 0.2$ as a function of nondimensional parameters, volume fraction (v_2), $\Lambda_{\mu} \stackrel{\text{def}}{=} \frac{\mu_2}{\mu_1}$ and $\Lambda_{\rho C} \stackrel{\text{def}}{=} \frac{\rho_2 C_2}{\rho_1 C_1}$. The surfaces give lubricant design alternatives. With decreasing Φ the design possibilities are reduced.

4. Summary

This work identified the properties of additives to introduce into base lubricants in order to reduce heat generation in fluid films. A model for the temperature rise in the fluid film that exists between an insulated bearing and housing was developed as a function of the rotation speed, the viscosity of the base lubricant and properties of the additives. The free design parameters for the additives are (1) their viscosities, (2) their mass densities, (3) their heat capacities and (4) their volume fraction. A series of simplifying assumptions allowed for a relatively simple model to be developed. Consequently,

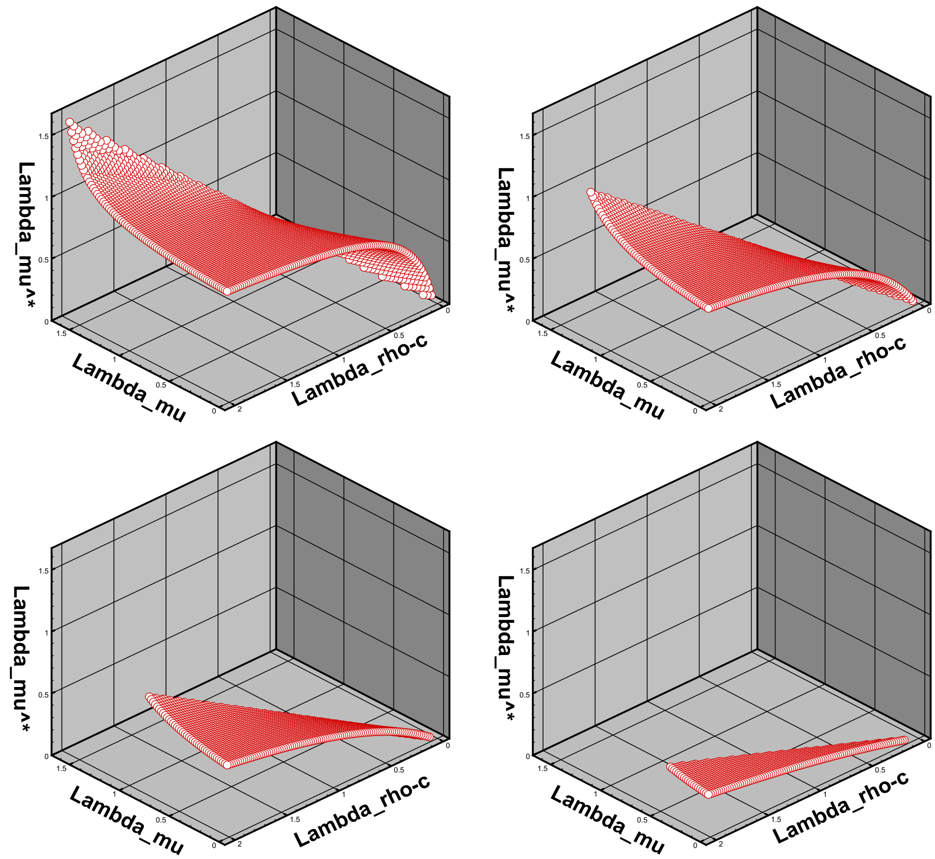

Fig. 3. Starting from left to right and top to bottom, the nondimensional surfaces that result for the volume fraction given by Eq. (3.6) for $\Phi = 0.8$, $\Phi = 0.6$, $\Phi = 0.4$ and $\Phi = 0.2$ as a function of nondimensional parameters $\Lambda_{\mu^*} \stackrel{\text{def}}{=} \frac{\mu^*}{\mu_1}$, $\Lambda_{\mu} \stackrel{\text{def}}{=} \frac{\mu_2}{\mu_1}$ and $\Lambda_{\rho C} \stackrel{\text{def}}{=} \frac{\rho_2 C_2}{\rho_1 C_1}$. The surfaces give lubricant design alternatives. With decreasing Φ the design possibilities are reduced.

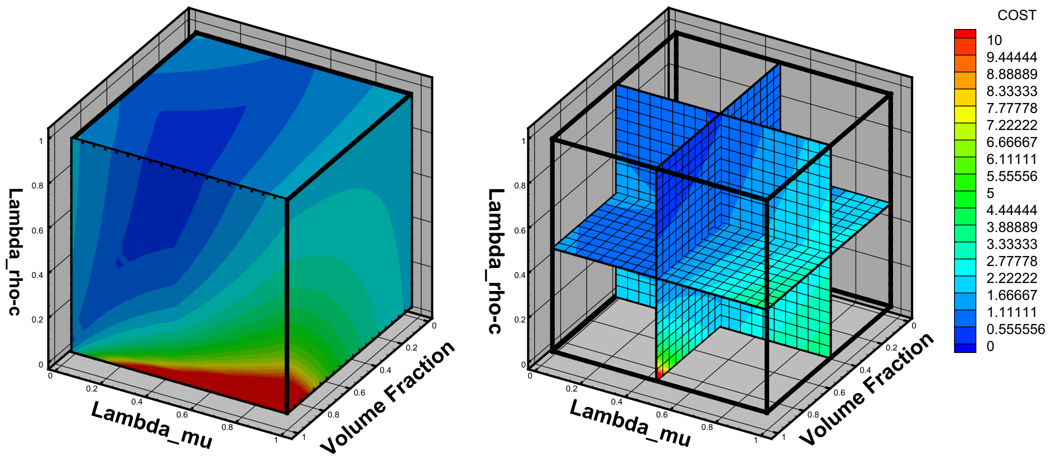

Fig. 4. The full nondimensional parameter space for the values of Π. Clearly, blue regions are advantageous and red regions are not. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

a relationship between the parameters was constructed, which yielded a surface in parameter space which could deliver a target value of heat generation reduction. Furthermore, due to the restriction that one would like to retain a certain effective overall viscosity, simultaneously, a cost function representing the simultaneous goals of heat reduction and acceptable effective viscosity, was developed. Current work is focussed on developing computational methods to (1) resolve time transient effects, including spatial variation in temperature and velocity fields around bearings and through the film thickness, (2) resolve the coupled problem of heat transfer to the journal bearing and housing and (3) track the changes in lubricant's properties as a function of the heat generated. A large-scale direct numerical method, based on a hybrid finite difference/discrete element method, has been previously developed by the author Zohdi (2007) for microscale particle-laden flows, and is being currently adapted to this problem, in order to investigate the points (1–3) raised above.

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